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Applications of Complex Numbers

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Abstract

The application of complex numbers is introduced to present the emerging growth in various fields of science and technology. Mainly two applications are specified: AC circuits and Fractals. A detailed study of fractals are presented. The common fractals are based on Juliet set and Mandelbrot set. The fractal analysis are used in astronomy, fluid mechanics, telecommunications, medicine, surface physics. The most useful use of fractals in computer science is the fractal image compression.

Keywords: Impedance, fractal, Juliet set, Mandelbrot set, fractal image compression.

Introduction

Complex numbers have essential concrete applications in a variety of scientific and related areas such as signal processing, control theory, electromagnetism, fluid dynamics, quantum mechanics, cartography and vibration analysis. The main applications are to ac circuits and complex pattern called fractals. Apart of complex numbers, complex functions are of great use in engineering and medicine. Using the complex plane, voltages across resistors, capacitors and inductors can be represented. The voltage across the resistor is regarded as a real quantity, while the voltage across an inductor is regarded as a positive imaginary quantity, and across a capacitor a negative imaginary quantity. The complex pattern fractals which are generated through iteration process have an immense applications in science, medicine and engineering. Some of them are discussed.

AC Circuits

Complex numbers are used a great deal in electronics. The main reason for this is they make the whole topic of analyzing and understanding alternating signals much easier. Using the complex plane, voltages across resistors, capacitors and inductors can be represented.The voltage across the resistor is regarded as a real quantity, while the voltage across an inductor is regarded as a positive imaginary quantity, and across a capacitor we have a negative imaginary quantity. The impedance of a circuit is the total effective resistance to the flow of current by a combination of the elements of the circuit. Symbol: Z Units: Ω

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The total voltage across all 3 elements (resistors, capacitors and inductors) is written as $\mathbf{V}_{RLC}$. Because V_L and V_C are considered to be imaginary quantities, Impedance $V_{RLC} = IZ$ with

$$Z = R + j(X_L - X_C)$$

Now, the magnitude (size, or absolute value) of Z is given by:

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

Angle θ represents the phase angle between the current and the voltage and is given by

$$\tan\theta = \frac{X_L - X_C}{R}$$

An alternating current is created by rotating a coil of wire through a magnetic field. If the angular velocity of the wire is ω, the capacitive reactance is given by

$$X_C = \frac{1}{\omega C}$$

Inductive reactance is given by: $X_L = \omega L$

Fractals

With computers, beautiful art can be generated from complex numbers. These designs are called fractals. Fractals are produced using an iteration process. Start with a number and then feed it into a formula. The result is substituted back into the formula, getting another result. And so on and so on...Thus a fractal is a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally self-similar and independent of scale. There are many mathematical structures that are fractals; e.g. Sierpinskitriangle, Koch snowflake, Peano curve, Mandelbrot set, and Lorenz attractor. Fractals also describe many real-world objects, such as clouds, mountains, turbulence, and coastlines that do not correspond to simple geometric shapes. Fractals start with a complex number. Each complex number produced gives a value for each pixel on the screen. The higher the number of iterations, the better the quality of the image. Common fractals are based on the Julia Set and the Mandelbrot Set [3][6].

Juliet Set

The Julia Set equation is:

$Z_{n+1} = (Z_n)^2 + c$

For the Julia Set, the value of c remains constant and the value of Z_n changes.

If we start with the complex number Z_1= 0.5 + 0.6j, and let c=0.3 and then feed this into the formula above, we have:

$Z_2 = (0.5 + 0.6j)^2 + 0.3 = 0.19 + 0.6j$

We now take this new answer and feed it back in:

$Z_3 = (0.19 + 0.6j)^2 + 0.3 = -0.0239 + 0.228j$

Continuing, we find that

$Z_4 = 0.24858721 − 0.0108984j$ and

$Z_5 = 0.3616768258 − 0.005418405698j$

The Mandelbrot Set

The Mandelbrot Set is the same as the Julia Set, but the value of c is allowed to change. The fractals have more and more applications in the science. The main reason is that they describe very often better the real world than traditional mathematics and physics. The other areas are in astronomy, computer science, fluid mechanics, telecommunication, surface physics and medicine.

Astronomy

Fractals will maybe revolutionize the way to see the universe. Cosmologists usually assume that matter is spread uniformly across space. But observation shows this is not true. Astronomers agree with that fact on "small" scales, but most of them think that the universe is smooth at very large scales. However, a dissident group of scientists claims that the structure of the universe is fractal at all scales. If this new theory is proved to be correct, even the big bang models should be adapted. But at present, cosmologists need more data about the matter distribution in the universe that we are living in a fractal universe.

Computer science

Actually, the most useful use of fractals in computer science is the fractal image compression. This kind of compression uses the fact that the real world is well described by fractal geometry. By this way, images are compressed much more than by usual ways (eg: JPEG or GIF file formats). Another advantage of fractal compression is that when the picture is enlarged, there is no pixelisation. The picture seems very often better when its size is increased. Fractal Image Compression:-Fractal compression is a compression method for digital images, based on fractals. The method is best suited for textures and natural images, relying on the fact that parts of an image often resemble other parts of the same image. Fractal algorithms convert these parts into mathematical data called "fractal codes" which are used to recreate the encoded image. The output images converge to the Sierpinski triangle [4]. This final image is called attractor for this photocopying machine. Any initial image will be transformed to the attractor if we repeatedly run the machine. On the other words, the attractor for this machine is always the same image without regardless of the initial image. This feature is one of the keys to the fractal image compression. Imagine a special type of photocopying machine that reduces the image to be copied by half and reproduces it three times on the copy. Several iterations of this process are on several input images. All the copies seem to converge to the same final image. Since the copying machine reduces the input image, any initial image placed on the copying machine will be reduced to a point as we repeatedly run the machine; in fact, it is only the position and the orientation of the copies that determines what the final image looks like.

Fluid mechanics

The study of turbulence in flows is very adapted to fractals. Turbulent flows are chaotic and very difficult to model correctly. A fractal representation of them helps engineers and physicists to better understand complex flows. Flames can also be simulated. Porous media have a very complex geometry and are well represented by fractal. This is actually used in petroleum science.

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Telecommunications

A new application is fractal-shaped antennae that reduce greatly the size and the weight of the antennas. Fractenna is the company which sells these antenna. A US company called Fractal Antenna Systems, Inc. makes antenna arrays that use fractal shapes to get superior performance characteristics, because they can be packed so close together. This design ensures performance improvements in antennae used in wireless, microwave, RFID (Radio Frequency Identification) and telecommunications. Fractal patterns can also be found in commercially available antennas, produced for applications such as cell phones and wifi systems by companies such as Fractenna in the US and Fractus in Europe. The self-similar structure of fractal antennas gives them the ability to receive and transmit over a range of frequencies, allowing powerful antennas to be made more compact [5].

Medicine

Biosensor interactions can be studied by using fractals. Modern medicine often involves examining systems in the body to determine if something is malfunctioning. Since the body is full of fractals, we can use fractal math to quantify, describe, diagnose and perhaps soon to help cure diseases. With modern imaging equipment such as CT scans and MRI machines, doctors can have access to a huge amount of digital data about a patient. Making sense of all the data can be time-consuming and difficult even for trained experts. Teaching computers to use mathematical processes to tell the difference between healthy lungs and lungs suffering from emphysema promises to help make faster, more reliable diagnoses. The fractal dimension of the lung appears to vary between healthy and sick lungs, potentially aiding in the automated detection of the disease. Cancer is another disease where fractal analysis helps diagnose and treat the condition. It is well known that cancerous tumors - abnormal, rapid growth of cells - often have a characteristic growth of new blood vessels that form a tangled mess instead of the neat, orderly fractal network of healthy blood vessels. Not only can these malfunctioning vessels directly harm the tissue, but they can also make it harder to treat the disease by preventing drugs from reaching into the inner parts of tumors where the drugs are most needed. Fractal analysis of cancer may be applied in many avenues, 'Tumors have a higher fractal dimension than normal tissues indicating their greater internal complexity'. Fractal dimensions have a tremendous buffering capacity, in that they grow in value even as the tumors themselves change little in terms of their apparent size. But once the dimension reaches a threshold value, the system changes radically, much as a ball traveling across a table drops when it reaches the edge, the power constants of the fractal dimensions - one incremental change gets them in trouble. If the genes that control that power function, the molecular changes, are understandable then whole new target for intervention can be made.

Surface physics

Fractals are used to describe the roughness of surfaces. A rough surface is characterized by a combination of two different fractals Computer graphics using CAD software is typically good art creating representations of man-made objects using primitives such as lines, rectangles, polygons, and curves in 2 D or boxes and surfaces in 3D [2]. These geometric primitives and usual tools for manipulating them typically prove inadequate when it comes to representing most objects found in nature such as clouds, trees, veins, waves, and a clump of mud. There has been considerable interest recently in chaos theory and fractal geometry as we find that many processes in the world can be accurately described using that theory. The computer graphics industry is rapidly incorporating these techniques to generate stunningly beautiful images as well as realistic natural looking structures. In what follows a description of a few of the more commonly used techniques will be given along with an example of each. It should be appreciated that usually the example is one from an image with a large if not infinite variation depending on the parameters, scale, and viewing position. As computers get smaller and faster, they generally produce more heat, which needs to be dissipated or else the computers will overheat and break. The smaller they are, the more this becomes a problem. Engineers at Oregon State University have developed fractal pattern that can be etched into a silicon chip to allow a cooling fluid (such as liquid nitrogen) to uniformly flow across the surface of the chip and keep it cool.

More Fractals

The "fractal fern" is generated completely by fractals. The fractal analysis is used in the realms of engineering, electronics, chemistry, medicine, even urban planning and public policy. But these illustrations just represent the beginnings of a revolution of understanding the functioning of the natural world. The fractals inspire more effective and efficient designs for our structures, devices, and systems. And no doubt, the most exciting applications of fractals are still in the future, waiting to be invented. Fractal Cities: Cities are complex systems that behave in some ways like living organisms. The rules of chaos theory and fractals apply directly to the evolution of cities, and the study of urban patterns allows to benefit from the experiments of past cultures to shape the future with as much awareness of the consequences of actions as possible. The real application of much of the above has arisen from attempts to model natural phenomena in the world we live in. Many of the mathematical techniques have found a firm place in the computer graphics industry as a means of creating both stunning graphical images as well as very natural looking structures. As the techniques become more standardised and more application areas are found they are likely to be incorporated as one of the standard tools iss CAD, painting and image processing software packages.

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