Original Article

ISSN: 2349 - 4891

International
Journal of Recent Research and Applied Studies
(Multidisciplinary Open Access Refereed e-Journal)

Modeling of Annual Maximum Monthly Rainfall using Generalized Extreme Value Distribution in Bangladesh

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Received 20th December 2018, Accepted 5th January 2019

Abstract

Generalized extreme value distribution was used in maximum monthly rainfall data of seven stations, Dhaka, Mymensingh, Chittagong, Comilla, Cox's Bazar, Maijdicourt and Rangamati in Bangladesh. The rainfall data for all stations over the period 1960-2012 was collected from Bangladesh Meteorological Department, Agargaon, Dhaka, Bangladesh. The statistical tools, descriptive statistics, Jarque-Bera test, Mann-Kendall test, Augmented Dickey-Fuller test, Kwiatkowski-Phillips-Schmidt-Shin test, Phillips-Perron test, generalized extreme value distribution, L-moment method; likelihood ratio test, chi-square test, probability plot, and density plot were used. The maximum monthly rainfall showed high fluctuations with minimum and maximum values in Comilla andMymensingh.The rainfall data displayed non-linear trend and stationary in nature and therefore stationary extreme value distribution was considered. On the basis of shape parameter, the appropriate distribution of Dhaka, Chittagong, Maijdicourt and Rangamati stations followed Gumbel distribution, Cox's Bazar station followed Weibull distribution, and Mymensingh and Comilla stations followed Frechet distribution. The return levels with 95% confidence interval for return period 5,10,50and100 years are estimated from stationary generalized extreme value distributionand found that the estimated return levels increase as the increase of return periods. The highest maximum monthly rainfall has found within 100 years for Dhaka, Comilla and Rangamati stations. But, Mymensingh, Chittagong, Cox-Bazar and Rangamati stations will need more than 100 years. These results are very useful for management of water by policy makers in Bangladesh.

Keywords: Maximum monthly rainfall, Generalized extreme value distribution, Gumbel distribution, Frechet distribution, Weibull distribution.

Introduction

The rainfall is considering one of the main natural resources in Bangladesh. Heavy rainfall is more harmful than too little rainfall, because heavy rainfall badly affects the agricultural crop, ecology and infrastructure, and causes untold troubles to human activities. It also causes damages of property and loss of lives. Every year in Bangladesh about 26,000 sq kmis flooded. On average,every year 95% of the total annual inflow (844,000 million cubic metre) of water into the country during the humid period (May to October) and only about 187,000 million cubic metre of stream flow is generated by rainfall (Banglapedia: Flood, 2006) [1].Bangladesh has been affected by flood many times but here we have mentioned some big floods, such as1786, 1794, 1822, 1825, 1838, 1853, 1864, 1865, 1867, 1871, 1876, 1879, 1885, 1890, 1900, 1902, 1904, 1954, 1955, 1962, 1966, 1968, 1969, 1974, 1987, 1988, 1989, 1993, 1998, 2000 (Banglapedia: Flood, 2006)[1].Usually, the heavy rainfall known as extreme or maximum rainfall and the study of maximum rainfall and known their patterns are very important to policy makers for policy implication.The statistical analysis of maximum rainfall has been done by many researchers in different locations in the world.Nadarajah and Choi (2007) analyzed annual maxima of daily rainfall for the years 1961-2001 are modeled for five locations in South Korea using generalizedextreme value distribution and found that the Gumbel distribution provided the most reasonable model for four of the five locations [2]. Feng et al. (2007) used the generalized extreme value distribution for modeling annual extreme precipitation from 1951-2000 at 651 weather stations in China and found that 12% of the stations have significant linear trends (non-stationary case) and rest (88%) of the stations does not shows any linear trend (stationary cases) [3].Shukla et al. (2010) have analysis annual extreme maximum rainfall in two locations in India using extreme value distribution and successfully concluded that generalized extreme value (GEV) distribution model is as the best fitted distribution model

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which satisfied the selection criteria, Anderson-Darling test (AD test or goodness of fit test) and Normality test (Q-Q plot) [4].Khamkong (2012) has analysis annual maximum monthly rainfall data from 1970-2010 for eight locations in the upper north Thailand and found thatall stations are stationary and one fitted the Weibull, the others seven stations fitted the Gumbel distributions[5].Varathan et al. (2010) used the daily extreme rainfall over the period 1900-2009 in Colombo for statistical modeling of generalized extreme value and generalized Pareto distributions and concluded that the Gumbel distribution provides the most appropriate model for the annual maximums of daily rainfall and the exponential distribution gives the reasonable model for the daily rainfall data over the threshold value of 100mm[6].Hasan et al. (2012) used the extreme temperature using 10 years with five different time periods are fitted to the generalized extreme value (GEV) distribution and found that three different time periods are non-normal that are significantly fitted to the GEV model[7].Zalina et al (2002) analyzed maximum rainfall for Malaysia using statistical eight distributions such as Gamma, Generalized Normal, Generalized Pareto, Generalized Extreme Value, Gumbel, Log Pearson Type III, Pearson Type III and Wake by and have been found that the GEV distribution is the most appropriate distribution for describing the annual maximum rainfall series in Malaysia [8].The environmental parameters are varying in region to region due to geographical locations. That’s way, the probability modeling of maximum rainfall for different locations in the world are necessary for management of water.Therefore, the purpose of the present study is to modelingof annual maximum monthly rainfall using the generalized extreme value distribution in Bangladesh.

Materials and Methods

Data Description

The daily rainfall of selected seven locations (Dhaka, Mymensingh, Chittagong, Comilla,Cox’s Bazar, Maijdicourtand Rangamati) was collected from Bangladesh meteorological department, Agargaon, Dhaka, Bangladesh over the range 1960-2012. The missing value of collected data was filling using by Linear Interpolation method. The daily rainfall data was convertedto monthly data as taking monthly total and thereafter takemonthly maximum rainfall within each year so the sample size became same as the number of years within study periods.

Table 1
Summary statistics of year extreme of monthly rainfall

Station→	Dhaka	Mymensingh	Chittagong	Comilla	Cox’s Bazar	Maijdicourt	Rangamati
Latitude	$23^0 76'$	$24^0 74'$	$22^0 26'$	$23^0 46'$	$21^0 44'$	$22^0 84'$	$22^0 65'$
Longitude	$90^0 38'$	$90^0 42'$	$91^0 81'$	$91^0 18'$	$91^0 97'$	$91^0 10'$	$92^0 18'$
Mean	141.25	159.09	216.49	136.04	200.51	184.81	177.25
Maximum	341.00	713.00	511.00	333.00	399.00	520.00	352.00
Minimum	56.00	17.00	80.00	72.00	45.00	82.00	62.00
Std. Dev.	58.78	104.35	78.51	50.24	60.53	76.88	68.24
Skewness	1.51	3.59	1.69	1.79	0.75	2.05	0.79
Kurtosis	5.86	18.25	6.71	6.60	5.37	8.96	3.23
Jarque-Bera (Prob.)	38.24 (0.00)	627.02 (0.00)	55.45 (0.00)	56.92 (0.00)	17.44 (0.00)	115.54 (0.00)	5.59 (0.00)

The latitude, longitude, descriptive statistics and normality test of annual maximum monthly rainfall for each selected stations are reported in Table 1. All of the distributions of selected stations are positively skewed, leptokurtic and non-normal. This Table also shows rainfall data have high flection among them but maximum variation occurs in Mymensingh station (std. deviation = 104.35) and minimum in Comilla station (std. deviation = 50.24).

Test of Trend

Mann (1945) [9] and Kendall (1975) [10] proposed a rank nonparametric test, known as Mann-Kendall test, and it is superior for detecting linear or non-linear trends (Hisdal et al. 2001 [11]; Wu et al. 2008 [12]). The null hypotheses (H_0) and alternative hypotheses (H_1) of Mann-Kendall test are equal to the non-existence and existence of a trend in the time series data, respectively. The Mann-Kendall statistic *S* is given asEq. (1)

$$S = \sum_{i=1}^{n-1} \sum_{=i+1}^{n} \operatorname{sgn}(x_j - x_i) \qquad (1)$$

The application of trend test is one to a time series$x_i$that is ranked from $i = 1, 2, ..., n-1$andx_j, which is ranked from $j = i+1, 2, ..., n$. Each of the data point x_i is taken as a reference point which is compared with the rest of the data points x_j so that,

$$S = \begin{cases} +1, & > (x_j - x_i) \\ 0, & = (x_j - x_i) \\ -1, & < x_j - x_i) \end{cases} \quad (2)$$

The statistic *S* is approximately normally distributed (when $n \geq 8$) with the meanand variance statistic defined as Eq. (3) and Eq. (4), respectively.

$$E(S) = 0 \quad (3)$$

$$Var(S) = \frac{n(n-1)(2n+5) - \sum_{i=1}^{m} t_i(i)(i-1)(2i+5)}{18} \qquad (4)$$

where, t_i is considered as the number of ties up to sample *i*. The test statistics Z_c is computed asEq. (5) and it follows a standard normal distribution.

$$Z_c = \begin{cases} \frac{S-1}{\sqrt{Var(S)}}, S > 0 \\ 0, S = 0 \\ \frac{S+1}{\sqrt{Var(S)}}, S < 0 \end{cases} \qquad (5)$$

A positive or negative value of Z_c signifies an upward or downward trend, respectively. A significance level α is also utilized for testing either an upward or downward monotone trend (a two-tailed test). If Z_c appears greater than $Z_{\alpha/2}$ where α depicts the significance level,

Test of Stationary

The stationarity test of annual maximum monthly rainfall is needed for choosing stationary or non-stationary extreme value distribution. The stationarity test conform using line graph and thewell-knownmethods such asAugmented Dickey Fuller (Dickey and Fuller 1979)[13], Kwiatkowski-Phillips-Schmidt-Shin (Kwiatkowski et al. 1992) [14] and Phillips-Perron (Phillips and Perron 1988)[15]. These methods are widely used in econometrics for checking stationarity.

Generalized Extreme Value Distribution

The annual maximum monthly rainfall is assume independently and identically distributed random variables for studying stochastic behavior using generalized extreme value distribution. Let, $X_1, X_2, \ldots, X_n$ are the random sample of size *n* from generalized extreme value distribution with cumulative distribution function (CDF)*F*(*x*) (Eq. (6)) is given by

$$F(x) = \exp\left\{ -\left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}} \right\} \quad (6)$$

defined on *x* such that $\left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right) > 0$ and with location parameter μ $(-\infty < \mu < \infty)$, scale parameter σ $(0 < \sigma < \infty)$ and shape parameter ξ

$(-\infty < \xi < \infty)$. If $\xi \to 0$ corresponds to the Gumbel distribution (Gumbel, 1958) [16], $\xi > 0$ to the Frechet distribution (Fréchet, 1927) [17] and $\xi < 0$ to the Weibull distribution (Weibull, 1939) [18]. The generalized extreme value distribution fitted mainly two cases such as stationary and non-stationary. Notice that stationary means the mean and variance are not change over time. If the study extreme data shows non-stationary in nature, we must take any function of time for location parameter or scale parameter or both to accurate modeling of generalized extreme value distribution using maximum likelihood methods otherwise we used *L*-moment method to estimate the parameters of generalized extreme value distribution. Experience indicates that the location and scale parameters being allowed to vary linearly and exponentially with time, respectively.The *L*-moments are expectations of certain linear combinations of order statistics and is a summary statistic for probability distributions and data samples. The *L*-moment method is not suitable for estimate the parameters of non-stationary process.

Test of hypothesis on Shape Parameter

The likelihood ratio (LR) test is used for testing the hypothesis of shape parameters as well as determines the best fit of model. Suppose L_0 and L_1 be the maximum likelihoods for reduced and full models, respectively. The likelihood ratio (LR) test statistic is denoted by λ and defined as

$$\lambda = -2\log\left(\frac{L_0}{L_1}\right) \qquad (7)$$

and it is distributed as chi-square distribution with degree of freedom equal to number of restriction imposes under null hypothesis.

Diagnostic Checking

There are two graphical tests is used for testing goodness of fit of selected modeland it is properly known as diagnostic checking. The graphical tests of goodness of fit are probability plot and density plot of the fitted models.

Return Period

The return level is the level that is expected on an average of once every *T* time period. The return level of rainfall can be approximated by the chi-square distribution. The amount of maximum rainfall is equal to return level and *T* corresponds to the year. Return levels are important for prediction and planning purposes and can be estimated from stationary models.

Results and Discussion

The annual maximum monthly rainfalls for Dhaka, Mymensingh, Chittagong, Comilla, Cox's Bazar, Maijdicourt and Rangamati stations in Bangladesh were considered as study variable. The rainfall data for all stations are shows non-normal and right skewed (skewness >0) distributions. Therefore, the generalized extreme value distribution modeling with annual maximum monthly rainfall over 1960-2012 is reasonable for this study.

Mann-Kendall test for annual maximum monthly rainfall of different stations are indicate that that the null hypothesis 'there is no trend in a series' are accepted (Table 2). The line graph (Figure 1) does not show any trend and any pattern of variation of annual maximum monthly rainfall has changed. So, the data of annual maximum monthly rainfall may be stationary but to conform of stationarity of data,formal tests (ADF, PP and KPSS) must be considered. The null hypothesis of ADF and PP are 'theannual maximum monthly rainfall data has unit root that is non-stationary' and the KPSS test is 'the annual maximum monthly rainfall data is stationary'. The ADF and PP tests imply that the null hypothesis is rejected and KPSS test imply that the null hypothesis is accepted for all stations (Table 3). Therefore, the all tests suggested that the annual maximum monthly rainfall data for all stations are stationary at level.Therefore,the generalized extreme value distribution in case of stationary process with no trend is apply to modeling of annual maximum monthly rainfall for selected stations. The parameters of this distribution must be estimate by *L*-moment methods.

Table 2
Mann-Kendall test for annual maximum monthly rainfall of different stations

Test	Dhaka	Mymensingh	Chittagong	Comilla	Cox's Bazar	Maijdicourt	Rangamati
Mann-Kendall (tau)	-0.188	0.017	-0.065	-0.133	-0.026	-0.001	0.047
p-value	0.050	0.866	0.500	0.163	0.788	1.00	0.623

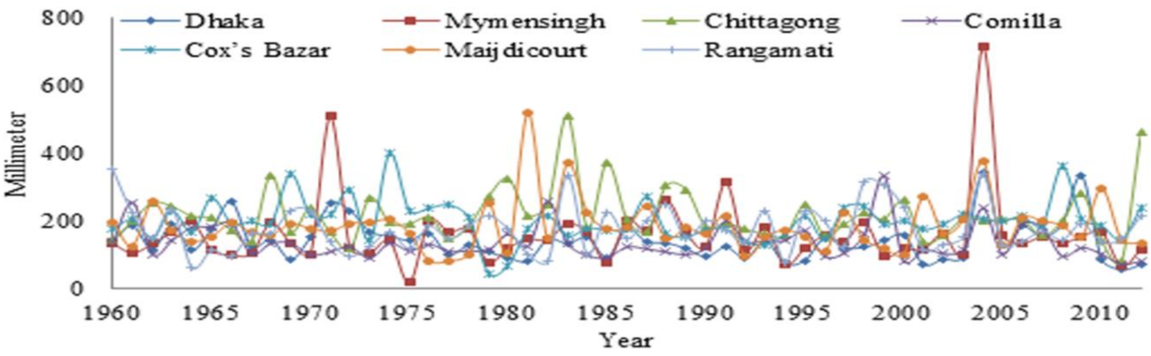

Figure I
Line graph for yearly extreme of monthly rainfall of all station

Table 3
Stationary test of rainfall for different station

	Dhaka	Mymensingh	Chittagong	Comilla	Cox's Bazar	Maijdicourt	Rangamati
ADF	-6.119		-7.326	-7.002	-6.484	-8.401	-7.888
(Prob.)	(0.000)	-7.978 (0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
PP	-6.098		-7.432	-6.996	-6.516	-8.304	-8.117
(Prob.)	(0.000)	-8.033 (0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
KPSS	0.181	0.086	0.106	0.096	0.135	0.060	0.104

Note: -2.919, -2.919 and 0.463 are 5% level critical values of ADF, PP and KPSS test, respectively.

The estimated parameters (μ , σ and ξ) of the generalized extreme value distribution under stationary process using by *L*-moment methods for the selected stations are presented in Table 4. The symbol '*se*' means standard error of estimated parameters is listed in brackets of Table 4.

Table 4
Models parameters estimated summary statistics for different station

Station	μ(se)	σ(*se*)	ξ(*se*)	- log L
Dhaka	114.186(6.188)	39.774(4.658)	0.096(0.107)	281.721
Mymensingh	121.015(7.986)	54.042(5.676)	0.101(0.061)	297.201
Chittagong	182.527(8.211)	54.716(5.852)	0.043(0.077)	296.818
Comilla	112.211(4.361)	28.260(3.491)	0.218(0.108)	267.475
Cox's Bazar	176.759(8.351)	56.450(5.405)	-0.152(0.057)	291.595
Maijdicourt	151.147(7.597)	49.728(5.621)	0.092(0.092)	293.219
Rangamati	146.726(8.488)	54.348(6.172)	-0.021(0.055)	294.949

The likelihood ratio (LR) tests for testing Gumbel distribution as $\xi = 0$ of the generalized extreme value distribution of different selected stations are recorded in Table 5. For Dhaka station, the value of likelihood ratio test is 0.9605 (Table 5) with associated probability is 0.3271, which is greater than 0.05 at 5% level of significance, hence we accept the null hypothesis that is $\xi = 0$. Therefore, the annual maximum monthly rainfall of Dhaka station follows Gumbel distribution. Again, for the Mymensingh station, the null hypothesis, $\xi = 0$ has been rejected (LR is 3.8787 and p-value is 0.0490 <0.050 in Table 5), and the value of ξ is 0.101 (Table 4) which is greater than zero. Therefore, the annual maximum monthly rainfall of Mymensingh station follows Frechet distribution. Similarly,for the Chittagong station, the null hypothesis is accepted (p-value is 0.5627 in Table 5)means the annual maximum monthly rainfall of Chittagong station follows Gumbel distribution. For the Comilla station, thenull hypothesis is rejected(p-value is 0.0222 in Table 5) means the annual maximum monthly rainfall of Comilla station does not follows Gumbel distribution and the value ξ is 0.218(Table 4), which is greater than zero hence the annual maximum monthly rainfall of Comilla station follows Frechet distribution. For the Cox's Bazar station, the null hypothesis is rejected (p-value is 0.0241 in Table 5) and the value of ξ is -0.152 (Table 4) means the annual maximum monthly rainfall of Cox's Bazar follows Weibull distribution. For the Maijdicourt station, the null hypothesis is accepted (p-value is 0.2594 in Table 5) means the annual maximum monthly rainfall of Maijdicourt station follows Gumbel distribution. Finally, for the Rangamati station, thenull hypothesis is accepted (p-value is 0.8519 in Table 5) means the annual maximum monthly rainfall of Rangamati station follows Gumbel distribution.

Table 5
Test of hypothesis of Gumbel distribution for different stations

Station	LR test	P- Value	Hypothesis and Comment	Best Distribution
Dhaka	0.9605	0.3271	$\xi = 0$ does not reject Gumbel hypothesis.	Gumbel
Mymensingh	3.8787	0.0490	$\xi = 0$ does not accept Gumbel hypothesis.	Frechet
Chittagong	0.3351	0.5627	$\xi = 0$ does not reject Gumbel hypothesis	Gumbel
Comilla	5.2307	0.0222	$\xi = 0$ does not accept Gumbel hypothesis	Frechet
Cox's Bazar	5.0860	0.0241	$\xi = 0$ does not accept Gumbel hypothesis	Weibull
Maijdicourt	1.2718	0.2594	$\xi = 0$ does not reject Gumbel hypothesis	Gumbel
Rangamati	0.0349	0.8519	$\xi = 0$ does not reject Gumbel hypothesis	Gumbel

The diagnostic check is very important for validity of the chosen model. There are several techniques are available for diagnostic check of fitted model but in present study, the probability plot and density plot were used. The probability plot and density plot for Dhaka, Mymensingh, Chittagong, Comilla, Cox's Bazar, Maijdicourt and Rangamati stations are presented Figure 2-8.The Figure 2-8 indicate that the probability plots for different stations are approximately straight line and the estimate density for all selected stations are approximately cover the peak of histogram. Hence, the chosen distributions for respective selected stations are fitted well.

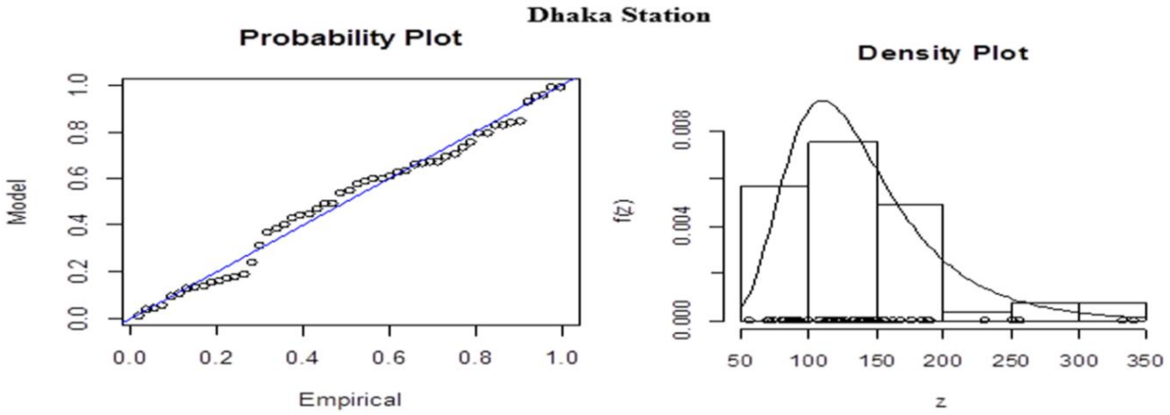

Figure II
Probability plot and density plot for Dhaka station

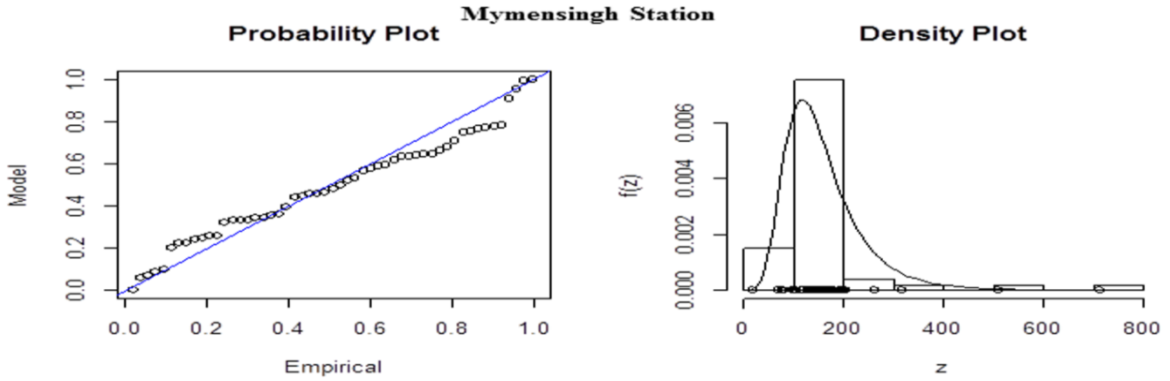

Figure III
Probability plot and density plot for Mymensingh station

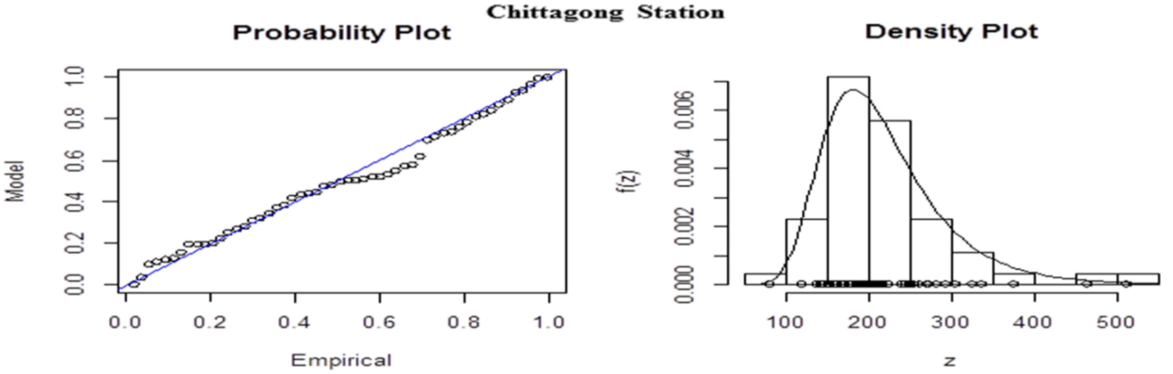

Figure IV
Probability plot and density plot for Chittagong station

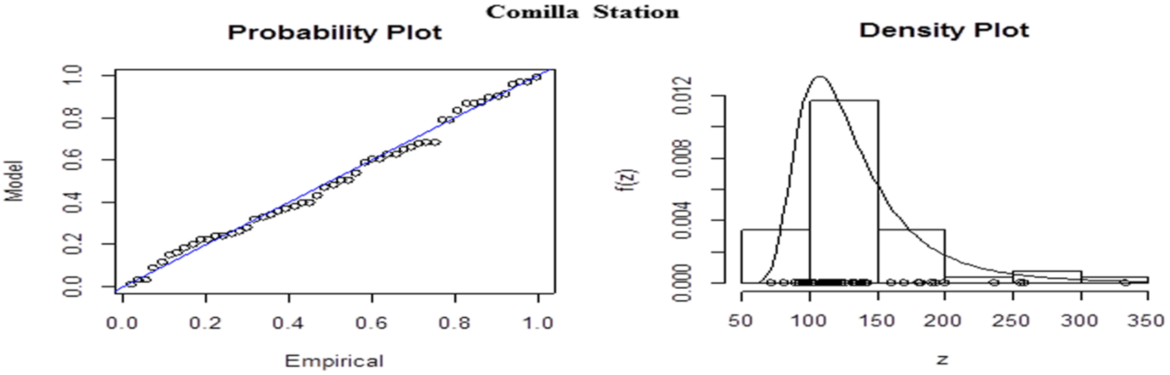

Figure V
Probability plot and density plot for Comilla station

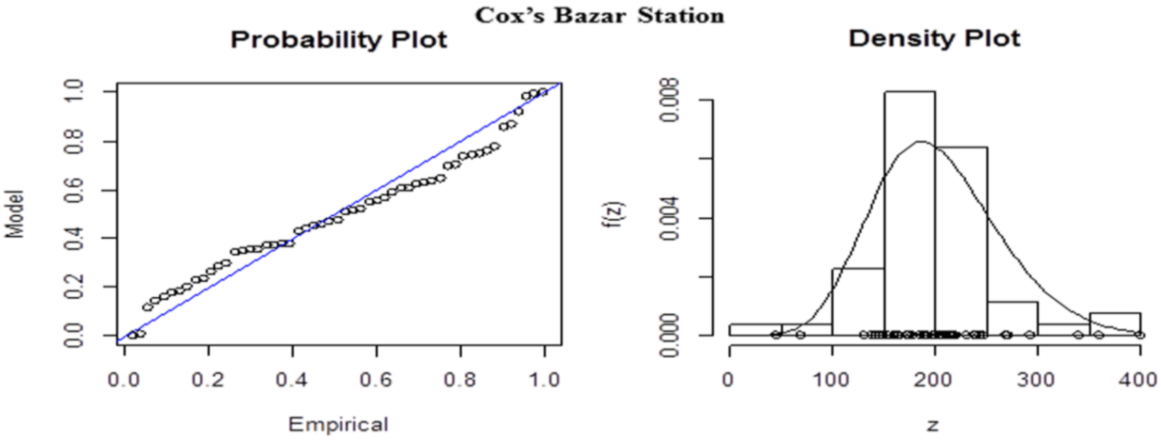

Figure VI
Probability plot and density plot for Cox's Bazar station

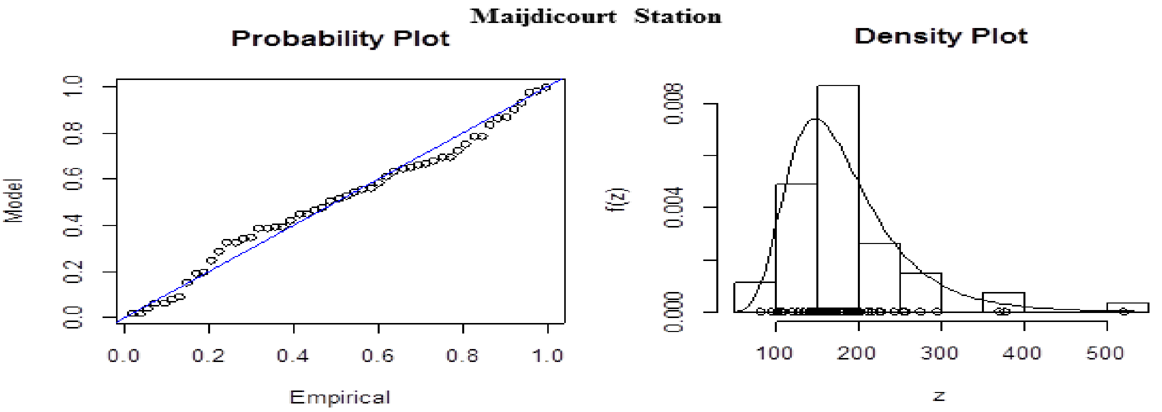

Figure VII
Probability plot and density plot for Maijdicourt station

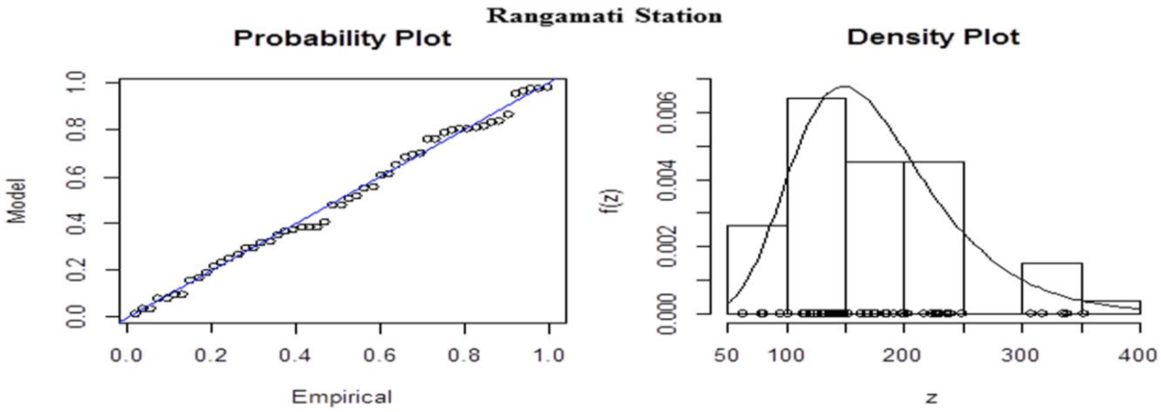

Figure VIII
Probability plot and density plot for Rangamati station

The prediction of probability that an annual maximum monthly rainfall will occurs in longer period is done on the basis of return level. The return levels are estimated by generalized extreme value distribution for stationary annual maximum monthly rainfall data of seven selected stations in Bangladesh. The return levels with 95% confidence interval for return period, T=5, T=10, T=50 and T=100 years are presented in Table 6. The estimated return levels increase as the increase of return periods (Table 6).The highest annual maximum monthly rainfall for Dhaka station over the period 1960-2012 is 341mm (Table 1). The annual maximum monthly rainfall for Dhaka station exceeds 341of the observation period for T=100 years (Table 6) considering point estimate. The highest annual maximum monthly rainfall data for Mymensingh station is 713(Table 1) over the periods 1960-2012. The annual maximum monthly rainfall which exceeds the values 713of the observation period is predicted to in more than T=100 years (Table 6). Similarly, the highest annual maximum monthly rainfall for Chittagong, Comilla, Cox’s Bazar, Maijdicourt and Rangamati are 511, 333, 399, 520 and 352 mm(Table 1) over the period 1960-2012, respectively.The annual maximum monthly rainfall of Chittagong, Comilla, Cox’s Bazar, Maijdicourt and Rangamati stations, which exceeds of the period are predicted to occurs in more than T=100, less than T=100, more than T=100, more than T=100 and less than T=100 years (Table 6), respectively.

Table 6
Return level for different stations

Station	T=5	T=10	T=50	T=100
Dhaka	178.34	214.07	302.37	344.11
	(158.69, 206.17)	(187.22, 263.46)	(246.93, 429.23)	(270.24, 523.28)
Mymensingh	208.52	257.53	379.38	437.31
	(182.60, 244.43)	(221.94, 314.64)	(310.41, 519.94)	(348.43, 626.05)
Chittagong	267.32	311.85	415.11	461.03
	(242.51, 301.21)	(279.06, 365.64)	(354.48, 542.92)	(383.88, 631.81)
Comilla	162.35	194.30	286.04	335.91
	(146.08, 187.19)	(169.83, 241.36)	(225.72, 424.48)	(250.33, 542.31)
Cox’s Bazar	252.48	284.37	342.96	363.65
	(232.83, 276.11)	(262.02, 315.45)	(313.16, 400.81)	(330.14, 432.70)
Maijdicourt	231.12	275.47	384.56	435.89
	(206.80, 265.08)	(242.39, 333.49)	(318.48, 530.12)	(349.19, 637.95)
Rangamati	226.99	266.21	350.41	385.14
	(203.58, 257.76)	(237.05, 317.12)	(298.42, 468.97)	(318.91, 545.41)

Conclusion

The generalized extreme value distribution is used to modeling of the annual maximum monthly rainfall of selected stations in Bangladesh. The annual maximum monthly rainfall data are indicates that there is no positive or negative trend. The line graph and stationarity test as ADF, PP and KPSS are shows that the annual maximum monthly rainfall data is stationary at level for all selected locations. By using the stationary case of generalized extreme value distribution, the best fitted models are the, Gumbel distributions for Dhaka, Chittagong, Maijdicourt and Rangamati stations. Cox’s Bazar station followed Weibull distribution, and Mymensingh and Comilla stations followed Frechet distribution respectively. The diagnostic check of fitted models for selected regions using probability plot and distribution plot that shows the fits are well. We provided the return periods of 5, 10, 50 and 100 years for each of respective locations. The highest annual maximum monthly rainfall will occur within 100 years for Dhaka, Comilla and Rangamati stations. But, Mymensingh, Chittagong, Cox-Bazar and Rangamati stations will need more than 100 years of that occurs. Since there is possibility of extreme rainfall will occur within next of 100 year for Dhaka, Comilla and Rangamati stations so the policy implication for these regions will be needed, immediately. To the best of our knowledge this article is the first to provide a statistical modeling of annual maximum monthly rainfall using generalized extreme value distribution in Bangladesh. Therefore, these results are very useful for policy makers to apply the policy for Bangladesh.

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