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Dominations of Hydrodynamic Model Based on an Ideal Fluid for Flat FRW Model

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Abstract

In this investigation we discussed the performance between Equation of State parameter (EoS) parameter for an ideal fluid and Hydrodynamic fluid with some consequences of FRW model. According to the recent observations of SNe-Ia supernova indicate that the expansion of the universe is accelerating due to an unknown form of energy which has large negative pressure known as Dark Energy. The dark energy is characterized by an Equation of State parameter $\omega = \frac{p}{\rho}$ where p and ρ are the pressure and energy density of an ideal fluid.

Keywords: FRW model, Hydrodynamic, EoS parameter.

Introduction

In this paper, we study a variety of fluid dynamics models as they arise in diverse application domains. This analysis allows us to investigate and provide insights to dynamic phenomena that arise in a variety of systems that share similar characteristics. Although, the Plasma Physics and Cosmology are two well-established fields of Theoretical Physics, the formulation of hydro and Magneto-hydrodynamics in space-time is a relatively new development.

One of the most important properties of FRW models is, as predicated by the inflation, the flatness, which agrees with observed cosmic microwave background radiation. Even through the universe on large scale, appears homogeneous and isotropic at the present time, there is no observational data that guarantee in an epoch prior to the recombination. In the early universe the sorts of matter fields are uncertain. The existence of anisotropy at early times is a natural phenomenon to investigate, as an attempt to clarify among other things, the local anisotropies that we observe today in galaxies, cluster and super clusters so at early time it appears appropriate to suppose a geometry that is more general than just the isotropy and homogeneous FRW geometry.

Thorne and Macdonald [1] gave a particularly good review of it. Other important works are due to Evans and Hawley [2], Sloan and Smarr [3], Zhang [4], Holcomb and Tajima [5], and later Holcomb [6]. Holcomb and Tajima [6] investigated linearized equations of motion for free photons, longitudinal and transverse oscillations, and Alfven waves in plasma at an ultra-relativistic temperature in a radiation-dominated Friedmann Robertson Walker (FRW) universe.

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In many astrophysical situations, the background space-time is neither pure ‘‘dust’’ nor pure radiation, but some kind of mixture of them. Also the temperature involved may be neither ultra relativistic nor non-relativistic. So it appears worthwhile to investigate the magneto hydrodynamics phenomena in a more general background metric and with a more general adiabatic equation of state for the plasma. In an attempt to extend the work of Holcomb and Tajima for a more general background with the metric components being an arbitrary power function of time.

Motivated by the situations discussed in above, in this paper we investigate the performance between EoS parameter for an ideal fluid and Hydrodynamic fluid with some consequences of FRW model. The outline of the paper is as follows:

Section 2, we provide the metric and field equations. Section 3, devotes to Hydrodynamic models. In section 4 we have provide the physical and kinematical properties of the model and finally in section 5 contain concluding remarks.

Metric and Field Equations

We consider the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) line element of the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\Phi^2)\right], \quad (1)$$

where the angle θ and ϕ are the usual azimuthal and polar angles of spherical coordinates, with $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. The coordinates (t, r, θ, ϕ) are called commoving coordinates. This means that the coordinate system follows the expansion of space, so that the space coordinates of objects which do not move with respect to the background remain the same. The homogeneity of the universe fixes a special frame of reference, the cosmic rest frame given by the above coordinate system. Also k is a constant representing the curvature of the space. The case of $k = 1$ corresponds to closed universe, flat universe obtained from $k = 0$ and the case of $k = -1$ corresponds to open universe. In view of above universe in this work we deliberate on the flat universe take after $k = 0$ with infinite radius. $R(t)$ represents the radius of the universe and the signature of the metric is $(+,-,-,-)$.

The energy momentum tensor for the source is

$$T_{ij} = (p + \rho)U_i U^j - p g_{ij}, \tag{2}$$

together with

$$U_i U^i = 1, \tag{3}$$

where U^i is the four velocity vector of the distribution, ρ is the energy density, p is the pressure,

Using equation (2) the components of T_i^j are as follows

$$T_1^1 = (\rho + p)U_1 U^1 - p\delta_1^1$$
$$= 0 - p$$
$$= -p.$$

Similarly $T_2^2 = T_3^3 = -p$, $T_4^4 = \rho$.

Therefore $T_1^1 = T_2^2 = T_3^3 = -p$ and $T_4^4 = \rho$. (4)

Also

$$T = T_1^1 + T_2^2 + T_3^3 + T_4^4$$
$$= \rho - 3p. \tag{5}$$

The field equation for gravitation is

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij}, \tag{6}$$

In the co-moving coordinate system the equation (6) for the metric (1), with the help of equation (4) takes the form

$$2\dot{H} + 2H^2 + KH^{\frac{2}{m}} = -(8\pi)p, \tag{7}$$

$$3H^2 + 3KH^{\frac{2}{m}} = (8\pi)\rho, \tag{8}$$

Hydrodynamic Model:

We have liberty to make some assumptions, since we have more unknowns with lesser number of field equations to determine them. For the complete determination of these unknowns, we use a special law of variation of Hubble parameter, proposed by Berman (1983) as

$$H = DR^{-m}, \tag{9}$$

where D and $m\,(\neq 0)$ are constants and

$$H = \frac{\dot{R}}{R}. \tag{10}$$

Solving equation (9), we get

$$R(t) = (at + b)^{\frac{1}{m}}, \tag{11}$$

where $a \neq 0$ and b are the constants of integration.

Physical and Kinematical Properties of Model:

In this section we discussed some kinematical and physical parameters of the model

a. Kinematical parameters:

The spatial volume,

$$V = (at + b)^{\frac{3}{m}}. \tag{12}$$

In our investigations we observed that the spatial volume *V* of the universe starts with big bang at $t \to 0$ and with the increase of time *t* it is always expands and increases, when $t \to \infty$ then spatial volume $V \to \infty$.

The scalar expansion,

$$\theta = \frac{3a}{m(at + b)}. \tag{13}$$

The generalized Hubble parameter,

$$H = \frac{a}{m(at + b)}. \tag{14}$$

From the equation (13) and (14) it is observed that the expansion scalar and the generalized Hubble parameter are the functions of cosmic time and the relation between them is $(H, \theta) \propto (1/t)$. This shows that the universe is expanding with the increase of cosmic time but the rate of expansion decrease to constant value which shows that the universe starts evolving with zero volume at initial epoch with an infinite rate of expansion.

The deceleration parameter,

$$q = -1 + m. \tag{15}$$

The sign of deceleration parameter shows that whether the model inflates or not. The negative sign of *q* indicates inflation and positive sign indicates deceleration. Also, recent observations of type Ia supernovae, expose that the present universe is accelerating and the value of deceleration parameter lies on some place in the range $-1 \le q \le 0$.

b. Physical Parameters:

Energy Density of Model

$$\rho = \left(\frac{1}{8\pi}\right)\left(\frac{2a^2}{m(at+b)^2} - \frac{2a^2}{m^2(at+b)^2} - \frac{k\left(a/m\right)^{\frac{2}{m}}}{(at+b)^{\frac{2}{m}}}\right) \tag{16}$$

In this model, we observe that at the initial epoch $t = 0$ the energy density is a decreasing function of time and attain a small constant value but vanishes at later times.

Pressure of model

$$p=\left(\frac{1}{8\pi}\right)\left(\frac{3a^2}{m^2(at+b)^2}+\frac{3k\left(a/m\right)^{\frac{2}{m}}}{(at+b)^{\frac{2}{m}}}\right). \qquad (17)$$

EoS parameter for an Ideal fluid

$$\omega=\frac{p}{\rho}=\frac{\left(\frac{3a^2}{m^2(at+b)^2}+\frac{3k\left(a/m\right)^{\frac{2}{m}}}{(at+b)^{\frac{2}{m}}}\right)}{\left(\frac{2a^2}{m(at+b)^2}-\frac{2a^2}{m^2(at+b)^2}-\frac{k\left(a/m\right)^{\frac{2}{m}}}{(at+b)^{\frac{2}{m}}}\right)}. \qquad (18)$$

Above equation represents the EoS parameter for an ideal fluid which is a function of time t. At the initial stage when the universe start to accelerate for small and whole interval of time the EoS parameter of the universe having value $\omega > 0$ (i.e. 1.5) which shows only matter dominated era i.e. our derive model with an ideal fluid is fully occupying the real matter and there is no chance other matters like DE etc.

EoS parameter for Hydrodynamic fluid

$$\omega=-1+\frac{1}{3}\frac{\left(\frac{4a^2}{m}-\frac{4a^2}{m^2}-\frac{2k\left(a/m\right)^{\frac{2}{m}}(at+b)^{\frac{1}{m}}}{m}\right)}{\left(\frac{-2a^2}{m}+\frac{2a^2}{m^2}+k\left(a/m\right)^{\frac{2}{m}}(at+b)^{\frac{-1}{m}}\right)}. \qquad (19)$$

Above equation (19) represents the EoS parameter for Hydrodynamic fluid which is a function of time t. At the initial stage when the universe start to accelerate for whole interval of time the EoS parameter of the universe having value $\omega < 0$ (i.e. -1.7) which shows only dark energy dominated era i.e. our derive model with Hydrodynamic fluid is fully occupying the Dark energy and there is no chance real matter.

Note that the time-dependent EoS parameter for an ideal fluid which is contrast with EoS parameter of hydrodynamic fluid, can justify the transition from the one regime to another (quintessence, phantom regime) as indicated by recent observations (Larson et al. 2011; Komatsu et al. 2011).

Conclusions

In this work, we investigate the performance between EoS parameter for an ideal fluid and Hydrodynamic fluid with some consequences of spatially homogeneous and isotropic flat Friedman Robertson Walker (FRW) cosmological model with perfect fluid. We find the solution of the field equations using constant deceleration parameter which yields the negative value of deceleration parameter. We found that the spatial volume is constant at $t \to 0$ and it expands exponentially as t increase and becomes infinitely large as $t \to \infty$. Also, we observe that the expansion scalar start with constant value at $t \to 0$ and then rapidly decreases up to a certain value after some finite time. The deceleration parameter q is negative and obtained a constant value -1, this range of deceleration parameter resembles with the observations of Type-Ia supernova, CMB radiations. The model with an ideal fluid is fully occupying the real matter and there is no chance of other matters.

References

1. K. S. Thorne and D. Macdonald, Mon. Not. R. Astron. Soc. 198, 339 ~1982.
2. C. R. Evans and J. F. Hawley, Astrophys. J. 332, 659 ~1988.
3. J. H. Sloan and L. Smarr, in *Numerical Astrophysics*, edited by J. M. Centrella, J. M. Le Blanc, and D. L. Bowers ~Jones and Bartlett, Boston, 1985.
4. X. H. Zhang, Phys. Rev. D. 39, 2933 ~1989.
5. K. A. Holcomb and T. Tajima, Phys. Rev. D. 40, 3809 ~1989.
6. K. A. Holcomb, Astrophys. J. 362, 381 ~1990.