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Theoretical Modeling of Double Stage Pulse Tube Refrigerator

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Abstract

The performance of the many devices like super conducting magnets, motors and infrared detectors and advanced electronic components greatly depends on the refrigerating system maintaining the required low temperature. Even though there are many forms of cryocoolers like, GM, Stirling, Pulse tube refrigerator etc, the pulse tube refrigerators are more attractive due to absence of moving parts at the low temperature region. In the present work a numerical model using the method of characteristics is developed for a double stage pulse tube. Here the total mass in a working cycle is divided in to a number of elements and the instantaneous properties of temperature and pressure are calculated for each element with respect to time and distance along the pulse tube. The specific refrigerating effect produced by each element and the total specific refrigerating effect is calculated.

Keywords: Cryogenics, Pulse tube refrigeration, Double stage pulse tube.

Introduction

Idealised model for a double stage pulse tube refrigerator has been developed. Method of characteristics has been used for developing the theoretical model. The total cold end mass of the pulse tube is divided in to a number of elements and the behaviour of each element inside the pulse tube has been found out using basic equations for the adiabatic process, assuming that the process inside the pulse tube is adiabatic.

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Theoretical Model

Here an idealised double stage pulse tube is taken for the analysis i.e. with no heat exchange between gas and tube wall, stratified gas flow in pulse tube and trapezoid pressure waves.

The following assumptions were made for the analysis

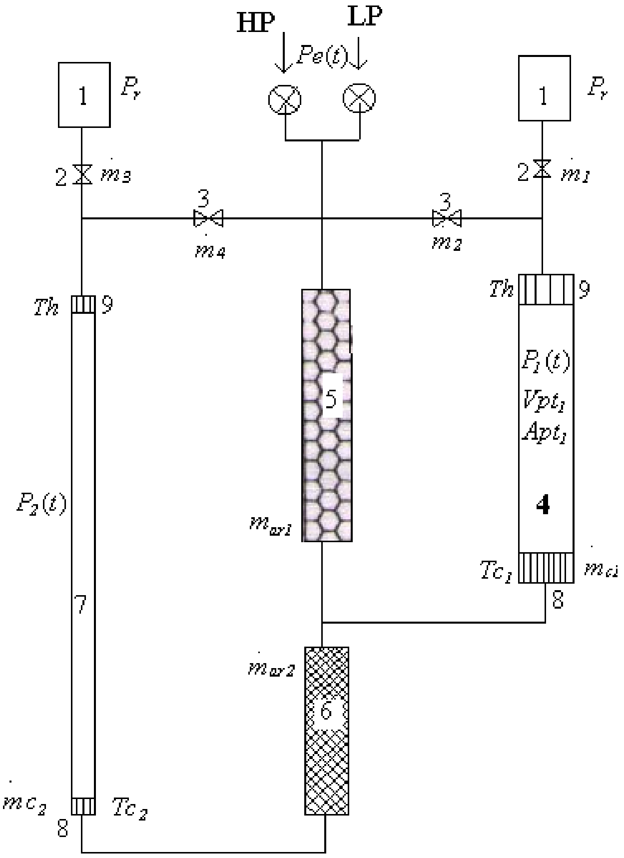
- Regenerator has perfect heat exchange i.e. no inefficiency or losses.
- The working fluid is regarded as ideal gas.
- Gas flow in the pulse tube is uniform, with no length wise mixing.
- The problem is one-dimensional.

Nomenclature

$\dot{m}$ =*mass flow rate in Kg/s*
Pe =external pressure,Pa
Pt =pulse tube pressure,Pa
T =temperature,K
β = *ratio of time of pressurisation or depressurisation to the time period*
Vpt =volume of the pulse tube, m^3
Ve = void volume of the regenerator, m^3
Pr= reservoir pressure, Pa
k= ratio of specific heats

Subscripts

1=fist stage
2=second stage
b=buffer
c=cold end
di=double inlet valve
hx=hot end heat exchanger
i=number of time element
j=no of gas element
n= n^{th} *time instant*



1. Buffer Volume
2. Orifice valve
3. Double inlet valve
4. First stage pulse tube
5. First stage regenerator
6. Second stage regenerator
7. Second stage pulse tube
8. Cold end heat exchanger
9. Hot end heat exchanger

Modeling of the first stage

The pressure at any instant‘t’ in the pulse tube is represented by $P_1(t)$

For the first stage the pressure in the pulse tube can be expressed as

$P_1(t) = Pl + (\frac{t}{\beta_1 \times \tau})(Ph - Pl)$ When $0<t<\beta_1 \tau$

$P_1(t) = Ph$ When $\beta_1 \tau<t<\tau/2$

$P_1(t) = Ph - \left(\frac{t-\tau/2}{\beta_1 \tau}\right) \times (Ph - Pl)$ When $\tau/2<=t<=(\beta_1 \tau+\tau/2)$

$P_1(t) = Pl$ When $(\beta_1 \tau+\tau/2)<t<\tau$

The orifices and the double inlet valves are regarded as impedance devices. With a certain degree of opening, the mass flow rates through the each valve are assumed to be proportional to the pressure difference imposed on its two sides. The mass flow rate through them can be written as

$$\dot{m} = C \times \Delta P_V$$

Where $\dot{m}$ is the mass flow rate through the valve and ΔP_V is the pressure difference across the valves. C is a constant which depends on the geometry of the valves and on the degree of opening of the valve. C is usually known as the conductance coefficient of the valve. For the first stage pulse tube the mass flow rate through the orifice is given by

$$\dot{m}_1 = C_1\left[P_1(t) - P_r(t)\right]$$

For the double inlet valve V_2 the mass flow rate is given by

$$\dot{m}_2 = C_2\left[Pe(t) - P_1(t)\right]$$

Where Pe(t) is the instantaneous value of the external pressure and C_2 is the conductance coefficient for the double inlet valve. The equation for the mass flow rate through the cold end for the first stage pulse tube can be developed as

$$\dot{m}_{c1} = \frac{Th}{Tc_1}\left(\dot{m}_1 - \dot{m}_2\right) + \frac{Vpt_1}{kRTc_1}\left[\frac{dP_1}{dt}\right] + \frac{Vdapt_1}{RTc_1}\left[\frac{dP_1}{dt}\right]$$

Where

$\dot{m}_{c1}$= mass flow rate through the cold end of the first stage

Vpt_1=Volume of the first stage pulse tube

$Vdapt_1$= dead volume between the hot end of the pulse tube and V_1 and V_2

The first half period, starting from the beginning of the gas inlet at the cold end of pulse tube is evenly divided into ‘n’ intervals of time. The gas that flows into the pulse tube at the interval j(1<= j<=n) is denoted by the gas element j. Mass of each element j is given by assuming the adiabatic compression of the working fluid inside the pulse tube which is represented by Δm_j.

$$\Delta m_j = \int_{tj-1}^{tj} \dot{m}_{C1}\, dt$$

Temperature of the gas element j at a time t_i is

$$T_j(t_i) = Tc\left[\frac{P_1(t_i)}{P_1(t_{j-1})}\right]^{K-1/K}$$

and volume of the element is given by

$$V_j(t_i) = \left[\frac{P_1(t_{j)}}{P_1(t_i)}\right]^{1/K} V_j t_j$$

Where

$$V_j(t_j) = \frac{\Delta m_j R}{P_1(t_j)} T_j(t_j)$$

$V_j(t_j)$ is the volume of the element when it enters in to the pulse tube. The position of the element j is defined as the distance between the cold end of the pulse tube and the gas elements upper side facing the hot end of the pulse tube. The position of the gas element j at a time t_i can be written as

$$X_j(t_i) = \frac{1}{Apt_1}\sum_{J=j}^{i} V_J(t_i)$$

$\sum_{J=j}^{i} V_J(t_i)$ represents the total volume of the gas elements between gas element j and the cold end of the pulse tube at the instant t_i. In the second half of the period gas element n leaves the cold end of the pulse tube at the time, t_{n+1}, and the $(n-1)^{th}$ element leaves the cold end when t is t_{n+2}. Generalising we can write the $(2n+1-i)^{th}$ element leaves the pulse tube at t_i, all the gas elements are out of the cold end of the pulse tube at the time t_{2n} and the cycle is finished. Now the position of the j^{th} element in the second half of the cycle can be written as

$$X_j(t_i) = \frac{1}{Apt_1}\sum_{J=j}^{2n-i} V_J(t_i)$$

Where A_{pt1} is the area of the first stage pulse tube.

The gas element j is completely out of pulse tube at a time t_{2n+1-i} with a temperature of

$$T_j\left(t_{2n+1-j}\right) = T_{C1}\left[\frac{P_1(t_{2n+1-j})}{P_1(t_{j-1})}\right]^{K-1/K}$$

In one cycle the element produces a refrigeration work of

$$Q_j = \Delta m_J C_p \left[T_{C1} - T_J(t_{2n+1-j})\right]$$

The specific cooling power obtained by each element is

$$q_{j1} = C_p \left[T_{C1} - T_J(t_{2n+1-j})\right]$$

Hence the total specific cooling power is

$$Q_{ref1} = f\sum_{J=1}^{n} q_{j1}$$

Specific cooling power is the refrigerating effect produced by an element having unit mass.

Modeling of the second stage

In the case of second stage the value of β will be greater than in the case of first stage. It is due to the fact that the gas flows in to the second stage is through the second regenerator which adds resistance to the flow of gas. This increases the pressurisation and depressurisation time inside the pulse tube. The equations for the second stage pulse tube are given as

Pressure variation

$$P_2(t) = Pl + \left(\frac{t}{\beta_2 \times \tau}\right)(Ph - Pl)$$ When $0<t<\beta_2 \tau$

$$P_2(t) = Ph$$ When $\beta_2 \tau<t<\tau/2$

$$P_2(t) = ph - \left(\frac{t-\tau/2}{\beta_2 \tau}\right) \times (Ph - Pl)$$ When $\tau/2<=t<=(\beta_2 \tau+\tau/2)$

$$P_2(t) = Pl$$ When $(\beta_2 \tau+\tau/2)<t<\tau$

Mass flow rate through the orifice valve

$$\dot{m}_3 = C_3\left[Pe(t) - P_2(t)\right]$$

Now for the double inlet valve V_4 the mass flow rate toward the pulse tube is defined as positive

$$\dot{m}_4 = C_4\left[P_2(t) - P_r(t)\right]$$

Mass flow rate through the cold end of the second stage can be developed as

$$\dot{m}c_2 = \frac{Th}{Tc_2}\left(\dot{m}_3 - \dot{m}_4\right) + \frac{Vpt_2}{kRTc_2}\left[\frac{dP_2}{dt}\right] + \frac{Vdapt_2}{RTc_2}\left[\frac{dP_2}{dt}\right]$$

Where

V_{dapt2}= dead volume between the hot end of the pulse tube and V_1 and V_2

V_{pt2} = Volume of the second stage pulse tube.

The discritisation procedure used in the first stage is also used for the second stage and the following equations were derived

The first half period, starting from the beginning of the gas inlet at the cold end of pulse tube is evenly divided into 'n' intervals of time. The gas that flows into the pulse tube at the interval j(1<= j<=n) is denoted by the gas element j,

Mass of each element j is given by

$$\Delta m_j = \int_{tj-1}^{tj} \dot{m}_{C2}\, dt$$

Temperature of the gas element j at a time t_i is

$$T_j(t_i) = Tc_2\left[\frac{P_2(t_i)}{P_2(t_{j-1})}\right]^{K-1/K}$$

and volume of the element is given by

$$V_j(t_i) = \left[\frac{P_2(t_{j)}}{P_2(t_i)}\right]^{1/K} V_j t_j$$

Where,

$$V_j(t_j) = \frac{\Delta m_j R}{P_2(t_j)} T_j(t_j)$$

The position of the j[th] element at a time t_i in the second stage pulse tube is

For the first half cycle

$$X_j(t_i) = \frac{1}{Apt_2} \sum_{J=j}^{i} V_J(t_i)$$

For the second half cycle

$$X_j(t_i) = \frac{1}{Apt_2} \sum_{J=j}^{2n-i} V_J(t_i)$$

The specific refrigeration power produced at the second stage of the double stage pulse tube is given by

$$Q_{ref\,2} = f \sum_{J=1}^{n} q_{j2}$$

Where $q_{j2} = C_p \left[T_{C2} - T_J(t_{2n+1-j})\right]$ is the refrigerating effect produced by each element having unit mass.

Results and Discussion

The equations were solved numerically by assigning following values to the parameters.

First Stage

Length of the pulse tube =200 mm
Inside diameter of the pulse tube =19mm
Length of the regenerator =163mm
Inside diameter of the regenerator =25 mm
Regenerator matrix used is stainless steel wire mesh
Buffer volume = $5X10^{-4}$ m^3
Porosity of the regenerator matrix = .7

Second Stage

Length of the pulse tube =350mm
Inside diameter of the pulse tube =14 mm
Length of the regenerator =190mm
Inside diameter of the regenerator =19mm
Buffer volume =$5X10^{-4}m^3$
Regenerator filling material is Er_3Ni
Porosity of the regenerator matrix =.7
Woking fluid is helium
Operating Frequency =1.6Hz
Density of the helium, ρ =2.389 kg/m^3
Ratio of specific heats, k =1.67
Characteristic gas constant for Helium(R) =2078.5 J/Kg K
Maximum pressure =22 .5x 10^5 N/m^2
Minimum pressure =6.8 x 10^5 N/m^2
Ambient temperature =300K
First stage cold end temperature =120K
Second stage cold end temperature =13.8K

Figure1 shows the mass flow rate through the cold end of the pulse tube for both first and second stage pulse tubes. It can see that that after the period of pressurisation the mass flow rate decreases considerably and becomes negligible. The depressurisation is taken as negative because the mass flow rate is in the opposite direction. From the figure it can be noticed that the mass flow rate in to the second stage is considerably higher when compared to the mass flow rate in the first stage and this is due to the fact that the second stage pulse tube is maintained at a considerably lower temperature when compared to the first stage.

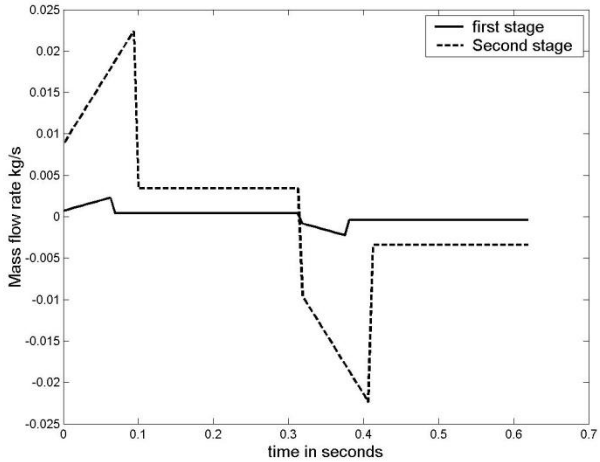


Fig. 1 Mass flow rate Vs time

Figure 2 represents the temperature variation with time for different elements. The graph shows the variations for the 1[st], 10[th] and 25[th] element. From the figure it can be seen that the first element enters in to the pulse tube at the beginning of cycle and is present inside the pulse tube till the end of the cycle. The first element enters in to the pulse tube at the cold end temperature 120K. The temperature of the first element increases from 120K to 195K during the process 1-2, due to adiabatic compression at the cold end. During the process 2-3 the temperature remains constant and during the expansion process the temperature decreases to 120k. This element actually does not contribute anything to the refrigerating effect since the initial and final temperature are the same.

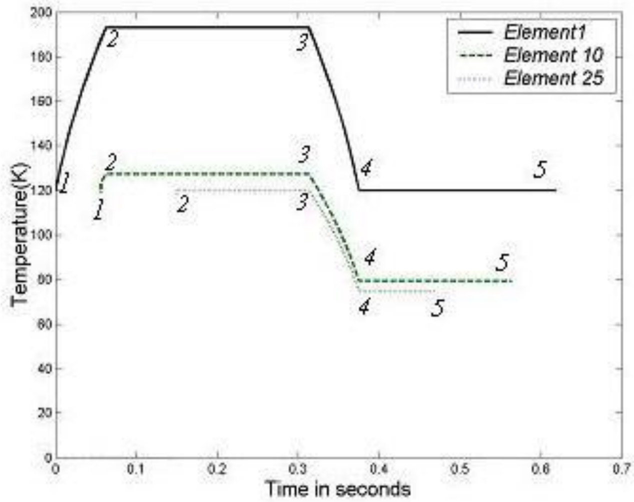


Fig. 2 Temperature Vs time graph for different elements (First Stage)

Now consider the path for the 10th element, which is entering at the cold end temperature, but for this element the compression region, is very low and resulting in only a small increase in temperature up to 130K, followed by the constant temperature region and expansion. After expansion, the temperature of this element reaches considerably lower than the cold end temperature which causes some refrigerating effect. The 25th element which enters after the compression region expands to the lower temperature directly from the cold end temperature. From the figure it can be seen that this element reaches the lowest temperature and produces the maximum refrigerating effect among the three elements considered.

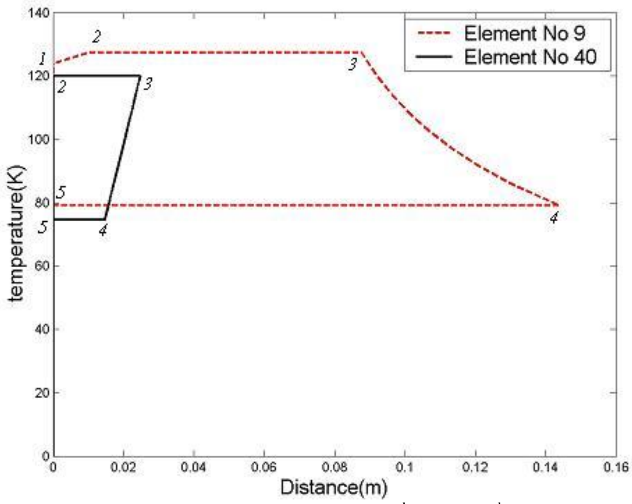


Fig. 3 Temperature Vs distance for 9th and 40th elements (first stage)

Figure 3 shows the temperature variation with respect to the distance. The graph is plotted for the 9th and 40th element for the first stage pulse tube. In process 1-2 (*Pl>Pt> Pb*) the gas enters the tube through both its cold end and hot end. Gas enters the tube at a pressure which is slightly higher than Pl due to the mass flow from the buffer to the tube. The gas element is compressed adiabatically and its temperature increases and moves towards the hot end of the pulse tube.

During the process 2-3 (*Pt=Ph*), the gas enters the cold end of the tube and leaves the hot end. When the element moves form the cold end to the hot end, the pressure in the tube remains constant. So the temperature of the element remains constant.

During the process 3-4, *(Ph<Pt<Pb)* the pressure in the tube decreases. Hence the element expands and its temperature decreases. The gas leaves from the cold end. After this process the temperature decreases to its lowest value. During the process 4-3, (*Pt=Pl*), the gas leaves the tube from the cold end and enters from the hot end. The pressure in the pulse tube remains constant while the gas element moves to the cold end and its temperature is constant.

Consider the 40th element which enters at a higher pressure, there is no compression, the element simply moves toward the hot end and expands to a lower temperature. The distance travelled by the gas elements are only a fraction of the total length of the pulse tube.

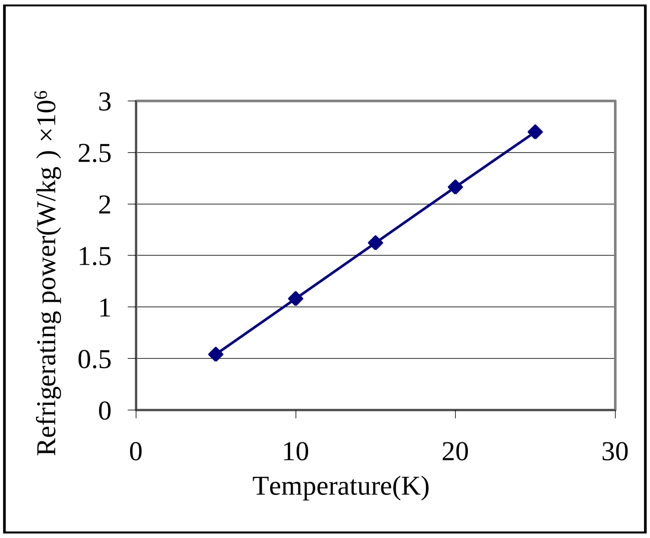


Fig. 4 Specific refrigerating effect Vs Cold end temperature (second stage)

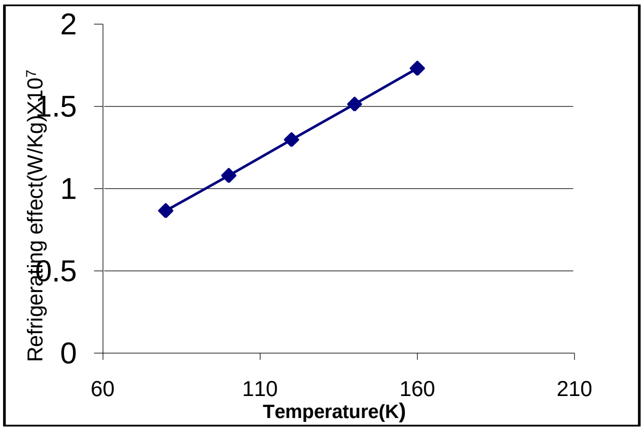


Fig. 5 Specific refrigerating effect Vs Cold end temperature (First Stage)

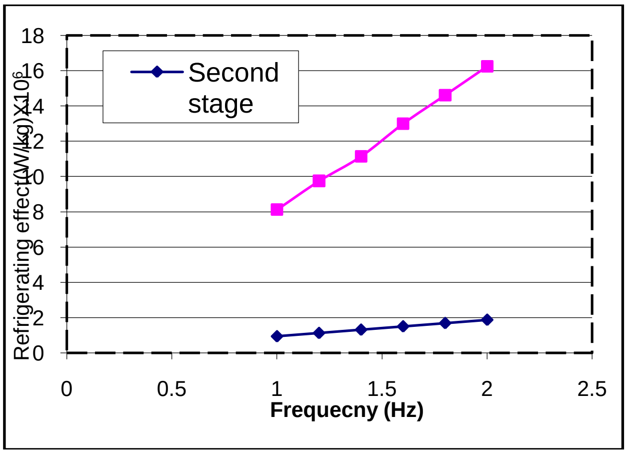


Fig. 6 Specific refrigerating effect Vs Frequency (first and second stage)

The variation of the specific refrigerating effect is shown in the figures 4, 5 and 6. It can be seen that the specific refrigerating effect increases with frequency. In this case the refrigerating effect per cycle remains constant but the total refrigerating effect increases because of the increase in the no of cycles per second. The specific refrigerating effect decreases with decrease in cold end temperature and finally becomes zero for a particular value of cold end temperature.

Conclusion

In this work a theoretical model for the double stage pulse tube refrigerator has been developed based on comprehensions of the internal processes. The simulated results give an insight to the basic phenomenon responsible for the cooling effect in double stage pulse tube refrigerator. The theoretical studies may be improved by incorporating real gas properties.

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