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A Study on Semi c(s)-Generalized Closed sets in Topological Spaces

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Abstract

In this paper, we have introduced a new class of closed set, as a weaker form of closed set namely semi c(s)-generalized closed set in topological space.

Keywords: scg-closed, sc*g-closed, sc(s) g-closed sets in topological space.
AMS Classification No. 54C05

1.1 STRONG AND WEAK FORMS OF OPEN SETS AND CLOSED SETS IN TOPOLOGICAL SPACES

Stone [STH] and Tong [TO_1] were investigated regular open sets and strong regular open sets, which are strong forms of open sets in topological spaces. Complements of regular open sets and strong regular open sets are called regular closed sets and strong regular closed sets respectively. Semi open set, a weak form of open set was introduced by Levine [LN_3]. Semi closed set was introduced by Biswas [BI_1]. Njastad [NJ], Levine [LN_4], Mashhour [MAD], Abd El- Monsef et al [MAD], Andrijevic [AD], Battacharyya and Lahiri [BL], Arya and Nour [AN_1], Maki et al [MDB_2], Pallaniappan et al [PR], Maki et al [MDB_1], Sundaram and Nagaveni [PSNA] and Pushpalatha [AP] have formulated α-closed sets, generalized closed sets, pre-closed sets, β-closed sets, semi generalized closed sets, generalized semi closed sets, αg-closed sets, regular generalized closed sets, generalized α-closed sets, weakly generalized closed sets, and strongly generalized closed sets, which are some weak forms of closed sets. Tong [TO_1] and Hatir et al [HNY] introduced B-sets and t-sets and α*-sets as weaker forms of closed sets. B-sets are weak forms of open sets. Sundaram [PS] introduced c-set and c(s)-set and Rajamani [MR] introduced c*-set. We recall the following definitions, which are used in this paper.

DEFINITION 1.1.1: A subset S of X is called a

a) regular closed [ST] if S = cl(int(S)) and regular open [ST] if S = int(cl(S)).
b) semi open [LN_3] if there exist an open set G such that $G\subseteq S\subseteq cl(G)$ and semi closed [BI_2] if there exist a closed set F such that $int(F)\subseteq S\subseteq F$. Equivalently, a subset S of X is called semi-open if $S\subseteq cl(int(S))$ and semi-closed if $S\supseteq int(cl(S))$[AD].
c) α-closed if $cl(int(cl(S)))\subseteq S$ and α-open if $S\subseteq int(cl(int(S)))$ [NJ].
d) pre-closed if $cl(int(S))\subseteq S$ and pre-open if $S\subseteq int(cl(S))$ [MAD].
e) β-closed [AE] (semi pre-closed [AN]) if $int(cl(S))) \subseteq S$ and a β-open [AE] (semi pre-open [AD]) if $S \subseteq cl(int(cl(S)))$.

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DEFINITION 1.1.2: For a subset S of X, the semi closure of S, denoted by scl(S), is defined as the intersection of all semi closed sets containing S in X and the semi interior of S, denoted by sint(S), is the union of all semi open sets contained in S in X [CH_1]. Pre closure [AD] of S, denoted by pcl(S), pre interior of S, denoted by pint(S), α-closure [NJ] of S, denoted by αcl(S), α-interior of S, denoted by αint(S), semi pre closure [AD] of S, denoted by spcl(S) and semi-pre interior of S, denoted by spint(S).

RESULT 1.1.3: For a subset S of X,

i) The semi closure is denoted by scl(S), defined as $scl(S) = S\cup int(cl(S))$ [AD]
ii) The semi interior is denoted by sint(S), defined as $sint(S) = S\cap cl(int(S)$ [AD]
iii) The pre closure is denoted by pcl(S), defined as $pcl(S) = S\cup cl(int(S))$ [AD]
iv) The pre interior is denoted by pint(S), defined as $pint(S) = S\cap int(cl(S)$ [AD]
v) The α-closure is denoted by αcl(S), defined as $\alpha cl(S)= S\cup cl(int(cl(S)))$ [NJ]

vi) The α-interior is denoted by αint(S), defined as αint(S)= S $\cap$ int(cl(int(S))) [NJ]
vii) The semi pre closure is denoted by spcl(S), defined as spcl(S)= S $\cup$ int(cl(int(S))) [AD]
viii)The semi pre interior is denoted by spint(S), defined as spint(S)=S $\cap$ cl(int(cl(S))) [AD]

DEFINITION 1.1.4: A subset A of X is called
i) generalized closed(briefly g-closed) set if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X[LN_4].
ii) semi generalized closed(briefly sg-closed) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in X [BL].
iii) generalized semi-closed (briefly gs-closed) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X [AN_2].
iv) generalized α-closed (briefly gα-closed) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in X [MDB_2].
v) α-generalized closed (briefly αg-closed) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X [MDB_1].
vi) generalized semi-pre closed(briefly gsp-closed) if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X [DO].
vii) regular generalized closed(briefly rg-closed) cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X [PR].
viii) weakly generalized closed(briefly wg-closed) cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is open in X [NA].
ix) strongly generalized closed (briefly strongly g-closed) cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X [AP].
x) semi c generalized-closed (briefly scg-closed) set if scl (A) $\subseteq$ U whenever, A $\subseteq$ U and U is c-set in X[RN].
xi) semi c* generalized-closed (briefly sc*g-closed) set if scl (A) $\subseteq$ U whenever, A $\subseteq$ U and U is c*-set in X[RN].

The complements of the above mentioned closed sets are their respective open sets.

DEFINTION 1.1.5: A subset S of X is called a
i) regular closed if S = cl(int(S)) [ST],
ii) t-set if int(S) = int(cl(S)) [TO_3],
iii) α*- set if int(A) = int(cl(int(A))) [GH_2],
iv) c-set if S = G $\cap$ F where G is open and F is α*-set in X [PS],
v) c*-set if S= G $\cap$ F where G is g-open and F is α*-set in X [PSMR],
vi) c(s)-set if S = G $\cap$ F where G is g-open and F is t-set in X [PS],

REMARK 1.1.6 : Every c-set in X is a c*-set in X [MR].

2. SEMI c(s)-GENERALIZED CLOSED SET IN TOPOLOGICAL SPACES

In 1970, Levine [LN_4] introduced the concept of generalized closed (briefly g-closed) sets in topological spaces and investigated some of their properties. Semi closed sets was introduced by Biswas [BI_1]. Nagaveni [NA], Pushpalatha [AP], Pallaniappan and Rao [PR] and Arya and Nour [AN_1] have introduced weakly generalized closed sets (wg-closed sets), strongly generalized closed sets (strongly g-closed sets), regular generalized closed sets (rg-closed sets) and generalized semi closed sets respectively. Tong [TO_1] and Hatir et al [HNY] introduced B-sets and t-sets and α*-sets are weaker forms of closed sets, α*-sets, t-sets and B-sets are weak forms of open sets. Sundaram [PS] introduced c-set and c(s) set and Rajamani [MR] introduced c*-set. We have introduced new class of set called sc(s)g-closed set in topological spaces and study some of their properties.

In this paper, we have introduced concept of semi c(s)-generalized closed set in topological spaces.

DEFINITION 2.1.1: A subset A of X is called a semi c(s)-generalized closed (briefly sc(s)-g closed) set if scl (A) $\subseteq$ U whenever, A $\subseteq$ U and U is c(s)-set in X. The complement of sc(s)-g closed set is called a sc(s)-g open set in topological spaces.

THEOREM 2.1.2: Every closed set in X is sc(s)-g closed in X but not conversely.

Proof: Assume that A is a closed set in X. Let U be a c(s)-set such that A $\subseteq$ U. Since A is closed, cl(A)=A. Therefore cl(A) $\subseteq$ U. Since scl(A) $\subseteq$ cl(A), scl(A) $\subseteq$ U. Hence A is sc(s)-g closed set in X.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.1.3: Consider the topological space X= {a, b, c} with topology τ = {φ, X, {a}, {a, b}}. The set {a, c} is sc(s)-g closed set but not closed set in X.

THEOREM 2.1.4: Every semi closed set in X is sc(s)-g closed set in X but not conversely.

Proof: Assume that A is a semi closed set. Let A $\subseteq$ U, U is a c(s)-set. Since scl(A) = A, scl(A) $\subseteq$ U. Therefore A is sc(s)-g closed set in X.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.1.5: Consider the topological space X= {a, b, c} with topology τ = {φ, X, {a}, {a, b}}. The set {a, c} is scg-closed set but not semi closed set in X.

THEOREM 2.1.6: Every sc*g-closed set in X is sc(s)-g closed set in X but not conversely.

Proof: Assume that A is sc*g-closed set in X. Let A $\subseteq$ G , where G is c(s)-set. Since every c(s)-set in X is a c*-set in X [MR], G is a c*-set and since A is sc*g-closed, scl(A) $\subseteq$ G. Therefore A is a sc(s)-g closed set.

The converse of the above theorem is need not be true as seen from the following example.

EXAMPLE 2.1.7: Consider the topological space X =

{a, b, c} with topology τ = { φ, X, {a, b}}. The set {a, c} is sc(s)-g closed set but not sc*g-closed set in X.

THEOREM 2.1.8: Every sc(s)-g closed set in X is gs-closed set in X but not conversely.

Proof: Assume that A is sc(s)-g closed set in X. Let A $\subseteq$U, where U is a c(s)-set, then U can be written as U = G$\cap X$, where G is g-open and X is t- set. Since A is sc(s)-g closed set. Therefore, scl(A) $\subseteq$G where G is open. Hence A is gs-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.1.9: Consider the topological space X= {a, b, c} with topology τ = { φ, X, {a}}. The set {a, c} is gs-closed set but not a sc(s)-g closed set in X.

REMARK 2.1.10: From the above results, we obtain the following diagram:

closed $\rightarrow$ semi closed $\rightarrow$ sc*g-closed $\rightarrow$ **sc(s)-g closed** $\rightarrow$ gs-closed

Figure 2.1.1

In the above diagram none of the implications can be reversed.

REMARK 2.1.11: The concept of sc(s)-g closed set is independent with the following classes of sets namely pre-closed, β-closed, gα-closed, wg-closed, g-closed, sg- closed, rg-closed, αg-closed and strongly g-closed sets in topological spaces.

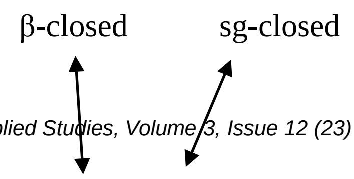
EXAMPLE 2.1.12: Consider the topological space X= {a, b, c} with topologies τ_1 = { φ, X, {a}} and τ_2 = { φ, X, {a, b}. In (X, τ_1) the set {a, b} is sc(s)-g closed set in X, but not pre-closed, β-closed, gα-closed and sg-closed set in X. In (X, τ_2) the set {b, c} is pre-closed, β-closed, gα-closed, and sg-closed set but not sc(s)-g closed set in X.

EXAMPLE 2.1.13: Consider the topological space X= {a, b, c} with topologies τ_1 = { φ, X, {a}, {a, b}} and τ_2 = { φ, X, {a}, {b, c}}. In (X, τ_1) the set {a, c} is sc*g-closed set in X, but not pre-closed, β-closed and sg-closed set in X. In (X, τ_2) the set {a, b} is pre-closed, β-closed and sg-closed set but not sc(s)-g closed set in X.

EXAMPLE 2.1.14: Consider the topological space X= {a, b, c} with topologies τ_1 = { φ, X, {a}, {a, b}} and τ_2 = { φ, X, {a, b}}. In (X, τ_1) the set {b} is both sc(s)-g closed and sc*g-closed set in X, but not g-closed, αg-closed and strongly g-closed set in X. In (X, τ_2) the set {b, c} is g-closed, αg-closed and strongly g-closed set but not sc(s)-g closed and sc*g-closed set in X.

EXAMPLE 2.1.15: Consider the topological space X= {a, b, c} with topologies τ_1 = { φ, X, {a}, {b}, {a, b}} and τ_2 = { φ, X, {a, b}}. In (X, τ_1) the set {b} is both scg-closed and sc*g-closed set in X, but not rg-closed set and wg-closed set in X. In (X, τ_2) the set {b, c} is rg-closed and wg-closed set but not scg-closed set in X.

REMARK 2.1.16: From the above discussion and known results we have the following diagram:



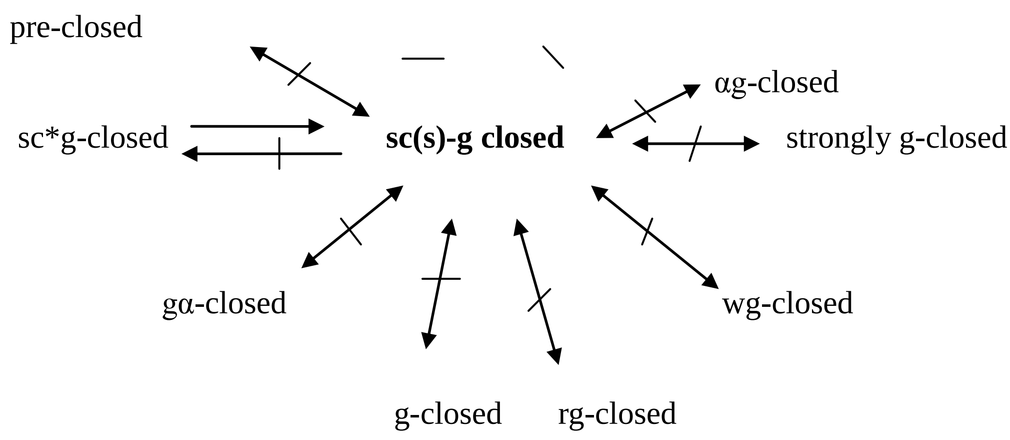


Figure 2.1.2

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List of Publications

1. scg, sc*g, sc(s)g–closed sets in topological spaces, *Proceedings of the 99th Session of the Indian Science Congress, Bhubaneswar* (2012), p. 89.
2. scg, sc*g, sc(s)g–continuous functions in topological spaces, *IJPAMS. ISSN: 0972-9828. Vol.1, Number 1 (2012)*, p. 65-72.
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4. A new class sc*g–set weaker form of closed sets in topological spaces, *IJCA(0975-8887), Vol. 55-No. 4, October (2012),* p. 25-29.
5. sc*g–Homeomorphisms in topological spaces, *IJERT, ISSN : 2278-0181, Vol.2, Issue 12 December (2013),* p. 3502-3503.
6. scg–closed sets in topological spaces, International Conference on "Mathematical Science and its Computational Applications (ICMSCA-2014)", *Dr. N.G.P. Arts and Science College, Kalapatti, Coimbatore, Tamil Nadu, August (2014),* p. 101-103.
7. An over-view of Topology and it's Applications, in the one day National Level Seminar "Modern Techniques and Applications in Mathematics", *Sree Saraswathi Thyagaraja College, Pollachi on 11th February, 2016.*
8. scg-Irresolute Function and sc*g-Irresolute Function in Topological Spaces, in one day National Conference on "Mathematical Modeling and Fuzzy Logic Applications" , *Department of Science & Humanities (Mathematics), RVS-Technical Campus, Coimbatore on 20th February, 2016.*