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Entropy Measure of Interval Valued Intuitionistic Fuzzy Values and It's Application to Multi Attribute Decision Making

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Abstract

In this paper an entropy measure for interval-valued intuitionistic fuzzy values are discussed. Also this paper presents a ranking for various alternatives using intuitionistic fuzzy weighted entropy. Finally a numerical example is illustrated to prove the effectiveness of the proposed method.

Keywords: Interval- valued intuitionistic fuzzy sets (IvIFS's), Entropy Measure, Multi Criteria Decision Making (MCDM).

Introduction

Multi Criteria Decision Making is concerned with structuring and solving decision and planning problems involving multiple criteria. In this paper, the information provided by the decision makers are represented as Interval valued Intuitionistic fuzzy numbers. The weights for evaluating criteria are calculated by means of entropy method. Entropy is an important notion to measure the uncertain information. The entropy was first studied by zadeh. Ye[4] proposed two entropy measures for IvIFS's and analyzed their problem in MADM problems. Xu and Yager[8] found some aggregation operators such as Intuitionistic Fuzzy Weighted Geometric operator and Intuitionistic Fuzzy Weighted operator. This paper is organized as follows: section 1 deals with the basic concepts of IvIFS's. Section 2 explains the proposed method. An numerical example is illustrated in section 3.

1 Basic Concepts

1.1 Interval-valued intuitionistic fuzzy sets:

Let a set Xbe fixed, an AIFS Ain $X$is defined as$A= \{(x,\ \mu A(x),\ \nu A(x)),x\in\ X\}$ where $\mu A$and $\nu A$are mappings from $X$to the closed interval [0,1] such that $0 \leqslant \mu A(x) \leqslant 1$, $0 \leqslant \nu A(x) \leqslant 1$ and $0 \leqslant \mu A(x)+\nu A(x) \leqslant 1$, for all $x\in X$, and they denote the degrees of membership and non-membership of element $x\in X$to set A, respectively.

The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x to A. Then for each x ∈ X, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals and their lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\nu_{AL}(x)$ and νAU (x), respectively, and thus we can replace Eq. with

$A = \{<x, [\mu_{AL}(x), \mu_{AU}(x)], [\nu_{AL}(x), \nu_{AU}(x)]>\}: x \in X\}$,

where $0 \leq \mu_{AU}(x) + \nu_{AU}(x)) \leq 1$ for any x ∈ X.

For convenience, Xu (2007a) called $\tilde{a}$= <[a, b], [c, d]>an interval-valued intuitionistic fuzzy number (IVIFN), where [a, b] ⊂ [0, 1], [c, d] ⊂ [0, 1] and b +d ≤ 1.

1.2 Score Function

Let ã= <[a, b], [c, d]> be an IVIFN, then the score function is defined as

$S(\tilde{a})= \frac{1}{2}(a-c+b-d)$, (1)

where s(ã) ∈ [−1, 1]. The larger the value of s(ã), the higher the IVIFN ã.

1.3 Accuracy Function

Let ã= <[a, b], [c, d]> be an IVIFN, then the accuracy function is defined as

$h(\tilde{a})= \frac{1}{2}(a+c+b+d)$, (2)

where h(ã)∈ [0, 1]. The larger the value of h(ã), the higher the accuracy degree of the IVIFN ã.

1.4 Hesitancy Degree

Let ã= <[a, b], [c, d]> be an IVIFN, then the hesitancy degree, the mid-point of intuitionistic fuzzy number is defined as

$\pi(\tilde{a})$ = [1-a-c,1-b-d] (3)

1.5 Comparison of two interval-valued intuitionistic fuzzy numbers

Let $\tilde{a}_1$=<[a_1, b_1], [c_1, d_1]> and $\tilde{a}_2$=<[a_2, b_2], [c_2, d_2]>be two interval valued intuitionistic fuzzy numbers. Let S($\tilde{a}_1$) and S($\tilde{a}_2$) denote the Score function of $\tilde{a}_1$ and $\tilde{a}_2$ respectively. Let H($\tilde{a}_1$) and H($\tilde{a}_2$) denote the accuracy

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functions of $\tilde{a}_1$ and $\tilde{a}_2$ respectively. Then ,

(i) If S ($\tilde{a}_1$)>S ($\tilde{a}_2$), then $\tilde{a}_1$ is greater than $\tilde{a}_2$, denoted by $\tilde{a}_1 > \tilde{a}_2$.

(ii) If S($\tilde{a}_1$)=S($\tilde{a}_2$), then

- If H ($\tilde{a}_1$) =H ($\tilde{a}_2$), then $\tilde{a}_1$ and $\tilde{a}_2$ represent the same information.
- If H ($\tilde{a}_1$) >H ($\tilde{a}_2$), then $\tilde{a}_1$is greater than $\tilde{a}_2$, denoted by $\tilde{a}_1 > \tilde{a}_2$.

1.6 Aggregation Operator of IVIFN's

Let $\tilde{a}_i$=<[a_i, b_i], [c_i, d_i]>, i=1,2,….n be a collection of interval-valued intuitionistic fuzzy values, and let IIWGA: $Q^n \rightarrow Q$ if,

$$\text{IIWGA}(\tilde{a}_1, \tilde{a}_2 \ldots \tilde{a}_n) = \left([\prod_{i=1}^{n} a_i^{w_i}, \prod_{i=1}^{n} b_i^{w_i}],\ [1-\prod_{i=1}^{n}(1-c_i)^{w_i}, 1-\prod_{i=1}^{n}(1-d_i)^{w_i}]\right) \quad (4)$$

Where w= $(w_1, w_2, \ldots\ldots w_n)^T$ be the weight vector of $\tilde{a}_i$ (i=1,2,…n) and w_i>0, $\sum_{i=1}^{n} w_i = 1$, then IIWGA is called the interval-valued intuitionistic fuzzy weighted arithmetic aggregation (IIWAA) operator.

1.7 Entropy Measure

Let ã= <[a, b], [c, d]> be an IVIFN. Motivated by the entropy proposed by [], in the following, an improved version of the entropy $E_N(\tilde{a})$ by incorporating the hesitancy degree$\pi(\tilde{a})$=[e,f] is given as follows:

$$E_N(\tilde{a}) = 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{|a-c|+|b-d|}{2+e+f+\min\{a+b, c+d\}} \quad (5)$$

1. Proposed Method

Denote n alternatives under consideration as s_1, s_2,……..s_n, the evaluation criteria as c_1, c_2,……c_n and the rating of each alternative s_j (j=1,2,……..n) with respect to criteria c_i (i=1,2,……..m) as s_{ij}.

For MCDM problem, let D denote the decision matrix provided by the decision maker, and w= $(w_1, w_2, \ldots w_n)^T$ be the weight vector , where $w_k \geq 0$, k=1,2,…..l and $\sum_{k=1}^{l} w_k = 1$.

Step 1: Construct the decision matrix.
Step2: Calculate the entropy measure of intuitionistic fuzzy values using equation (5).
Step3: Obtain weight vector for each criteria.
Step4: Aggregate IVIFN's using equation (4).
Step5: Find Score for each alternative by equation (1).
Step6: Rank all the alternatives. The alternative with highest score is selected to be the best alternative.

2. Numerical Example

A firm needs to identify a best supplier from a set of three suppliers namely S_1, S_2, S_3. Three criteria must be evaluated. They are Quality (C_1), Reliability (C_2), and Price (C_3), on time delivery (C_4). The interval valued intuitionistic decision matrix provided by the decision maker is given below

Step 1: Construct decision matrix.

$$D = \begin{matrix} & s_1 & s_2 & s_3 \\ C_1 & ([0.4,0.8],[0,0.1]) & ([0.3,0.5],[0.2,0.3]) & ([0.7,0.8],[0.1,0.2]) \\ C_2 & ([0.3,0.6],[0.3,0.4]) & ([0.3,0.6],[0.1,0.3]) & ([0.4,0.5],[0.3,0.5]) \\ C_3 & ([0.2,0.5],[0.1,0.4]) & ([0.5,0.2],[0.1,0.4]) & ([0.2,0.7],[0.2,0.3]) \\ C_4 & ([0.4,0.7],[0.1,0.2]) & ([0.2,0.3],[0.3,0.4]) & ([0.4,0.7],[0.1,0.2]) \end{matrix}$$

Step2: Calculate the entropy measure of intuitionistic fuzzy values

By using equation (5), entropy of the interval valued intuitionistic fuzzy values is resulted in the decision matrix as below

$$E_N(D) = \begin{matrix} & s_1 & s_2 & s_3 \\ C_1 & 0.3929 & 0.9063 & 0.52 \\ C_2 & 0.9355 & 0.8387 & 0.9677 \\ C_3 & 0.9394 & 0.8235 & 0.8709 \\ C_4 & 0.8966 & 0.9394 & 0.7241 \end{matrix}$$

Step3: Obtain weight vector for each criteria.

The value of weight vector for the evaluated criteria is obtained as
min E= $1.8192w_1+2.7419w_2+2.6338w_3+2.5601w_4$
such that

$0 \leq w_1 \leq 0.3$, $0.1 \leq w_2 \leq 0.2$
$0.2 \leq w_3 \leq 0.5$, $0.1 \leq w_4 \leq 0.3$, and $w_1+w_2+w_3+w_4=1$.

This problem can be solved using linear programming manually calculation and the results obtained are w_1=0.3,w_2=0.3,w_3=0.1,w_4=0.3.

Step4: Aggregate IVIFN's using equation (4).

A_1= <[0.342,0.673],[0.139,0.261]>
A_2= <[0.280,0.413],[0.194,0.342]>
A_3=<[0.441,0.659],[0.175,0.314]>

Step5: Find Score for each alternative by equation (1)

S(A_1)= 0.3075
S(A_2)=0.0785
S(A_3)=0.3055

Step6: Rank all the alternatives

The optimal ranking order of the alternatives is $A_1 > A_3 > A_2$.

Therefore the best alternative is A_1.

2. Conclusion

Thus in this paper weighted entropy measure and intuitionistic fuzzy weighted geometric operator have been used to rank the alternatives. From the findings, it shows that the interval valued intuitionistic fuzzy set is a suitable tool to solve the uncertainty and fuzziness in the multiple criteria decision making problem.

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