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A Two Warehouse Inventory Model with Exponential Demand and Constant Deterioration Rate

U. B. Gothi[1], Dishanshi Joshi[2] & Kirtan Parmar[3]

[1]*Head & Associate Professor., Dept. of Statistics, St. Xavier's College (Autonomous), Ahmedabad, Gujarat, India.*
[2]*Research Scholar, Dept. of Statistics, St. Xavier's College (Autonomous), Ahmedabad, Gujarat, India.*
[3]*Adhyapak Sahayak, Dept. of Statistics, St. Xavier's College (Autonomous), Ahmedabad, Gujarat, India.*

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Abstract

In this paper, we have analysed a two-warehouse inventory model for deteriorating items having exponential demand pattern and time varying holding cost. Here we have considered two different deterioration rates for two different warehouses. In the model considered here, shortages are allowed and partially backlogged. Numerical example is provided to illustrate the model and sensitivity analysis is also carried out for the parameters.

Keywords: Warehouse Inventory, Exponential, Deterioration.

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I. Introduction

We know that inventory model means mathematical equation or formula that helps a firm in determining the economic order quantity and the frequency of order to keeps goods or services flowing to the customer without interruption or delay. In many real life experiences, some customers wait for an item during shortage period, some customers wait until replenishment of a product if there is short amount of waiting time. However, some customers are impatient hence they will look for other options. Researchers have developed economic order quantity models that focused on deteriorating items with time varying demand and shortages. Donaldson [8] developed an optimal algorithm for solving classical no shortage inventory model analytically with linear trend in demand over fixed time horizon. Dave U. [7] proposed a deterministic lot size inventory model with shortages and a linear trend in demand.

Goswami and Chaudhuri [9] discussed different types of inventory models with linear trend in demand. Hartley [11] mentioned some basic information about two-warehouse model. In this he includes the value of transporting a unit from RW to posses OW was thought about earlier. Sarma [20] extended the transportation value in Hartely's Model. In this model, the case of finite refilling rate is extended by Murdeshwar and Sathe [17]. Dave [6] extended the case of bulk unleash pattern for each finite and infinite refilling rates and he also gave the answer for Sarma's [20] Model. In the two warehouse model for deteriorating things and shortage, finite refilling rate is given by Pakkala and Achary [18]. After that, Goswami and Chaudhuri [9] Benkherouf [2] and Kar Bhunia and Malti [14] gave the concept of time varying demand and stock dependent demand. Buzacott [4] and Misra [16] had developed EOQ model with constant demand and one rate of inflation for all associated prices. Yang [21] gave a two warehouse inventory model for one item with some constant demand and shortages under inflation. Stock dependent demand is studied by Zhou and Yang [22]. He also extended the partial backlogging and then compared with two warehouse models supported the minimum price approach. In this model, Jaggi et al. [13] had consulted about the optimum inventory replacement policy for deteriorating things under inflationary conditions. A settled inventory model for deteriorating things with two warehouse by minimizing cyberspace gift price of the entire price is developed by Hsieh et al. [12]. Kumar et al. [15] gave an inventory model with time dependent demand and limited storage facility under inflation. Hariga [10] studied the effects of inflation and time value of money on an inventory model with time dependent demand rate and shortages. Therefore, the demand of the product during its growth and decline phases can be well approximated by continuous time dependent function such as exponential or linear.

Recently, Kirtan Parmar and U. B. Gothi [19] have developed an economic order quantity (EOQ) model with constant deterioration rate and time-dependent demand and inventory holding cost. Devyani Chatterji and U. B. Gothi [5] have developed three-parametric Weibully deteriorated EOQ model with price dependent demand and shortages under fully backlogged condition. Ankit Bhojak and U. B. Gothi [3] have developed an inventory model for ameliorating and deteriorating items with time dependent demand.

Correspondence
Dishanshi Joshi
E-mail: joshidishanshi@gmail.com, Ph. +9197242 10783

Ajay Singh Yadav and Anupam Swami [1] have developed a two-warehouses inventory model in which they have assumed exponential demand. They have taken different inventory holding costs in both OW and RW and profit maximization technique is used.

In this paper, we have developed a deterministic inventory model for decaying items with two warehouses. Here, we assumed that inventory costs in RW are higher than those in OW, when demand rate is an exponentially increasing with time. Replenishment rate is taken as infinite and lead time is zero. Holding cost varies with time and shortages are allowed in OW.

II. Notations

1. $I_1(t)$: Inventory level for the rented warehouse (RW).
2. $I_2(t)$: Inventory level for the owned warehouse (OW).
3. $I_3(t)$: Inventory level for the backorder.
4. w : The capacity of the owned warehouse.
5. R(t) : Demand rate.
6. $\theta(t)$: Rate of deterioration per unit time.
7. δ : The backlogging rate ($0 < \delta < 1$).
8. A : Ordering cost per order during the cycle period.
9. C_d : Deterioration cost per unit per unit time.
10. C_h : Inventory holding cost per unit / unit time.
11. Cs : Shortage cost due to lost sales per unit.
12. l : Opportunity cost due to lost sales per unit.
13. IM : The max inventory level during [0, T].
14. IB : The maximum inventory level during shortage period.
15. Q : Order quantity in one cycle.
16. p_c : Purchase cost per unit.
17. t_1 : The time at which the inventory level reaches zero in RW ($t_1 \geq 0$).
18. t_2 : The time at which the inventory level becomes zero in OW ($t_2 \geq 0$).
19. T : The length of cycle time.
20. TC : Total cost per unit time.

III. Assumptions

1. The demand rate of the product is $R = Ae^{\lambda t}$ ($\lambda > 0$).
2. Time to deteriorate of an item follows the Exponential distribution with p.d.f.
$$g(t) = \begin{cases} \theta e^{-\theta t} & ; t > 0 \\ 0 & ; \text{otherwise.} \end{cases} \quad (0 < \theta < 1)$$
where θ is the deterioration rate and $\theta = \theta_1$ for RW & $\theta = \theta_2$ for OW.
3. Holding cost is a linear function of time and it is $C_h = a_2+b_2t$ in RW ($a_2, b_2 > 0$) and $C_h = a_1+b_1t$ in OW ($a_1, b_1 > 0$).
4. The OW has fixed capacity 'w' and RW has unlimited capacity.
5. First the units kept in RW are used and then of OW.
6. Replenishment rate is infinite and instantaneous.
7. Shortages occur and they are partially backlogged.
8. Repair or replacement of the deteriorated items does not take place during a given cycle.

IV. Mathematical Model and Analysis

At time t = 0 the inventory level is S units. From these 'w' units are kept in owned warehouse (OW) and rest in the rented warehouse (RW). The units kept in rented warehouse (RW) are consumed first and then of owned warehouse (OW). Due to the market demand and deterioration of the items, the inventory level decreases during the period $[0, t_1]$ and the inventory in RW reaches to zero. Again with the same effects, the inventory level decreases during the period $[t_1, t_2]$ and the inventory in OW will also become zero. Thereafter, shortages are allowed to occur during the time interval $[t_2, T]$, and all of the demand during the period $[t_2, T]$ is partially backlogged. The pictorial presentation is shown in the Figure – 1.

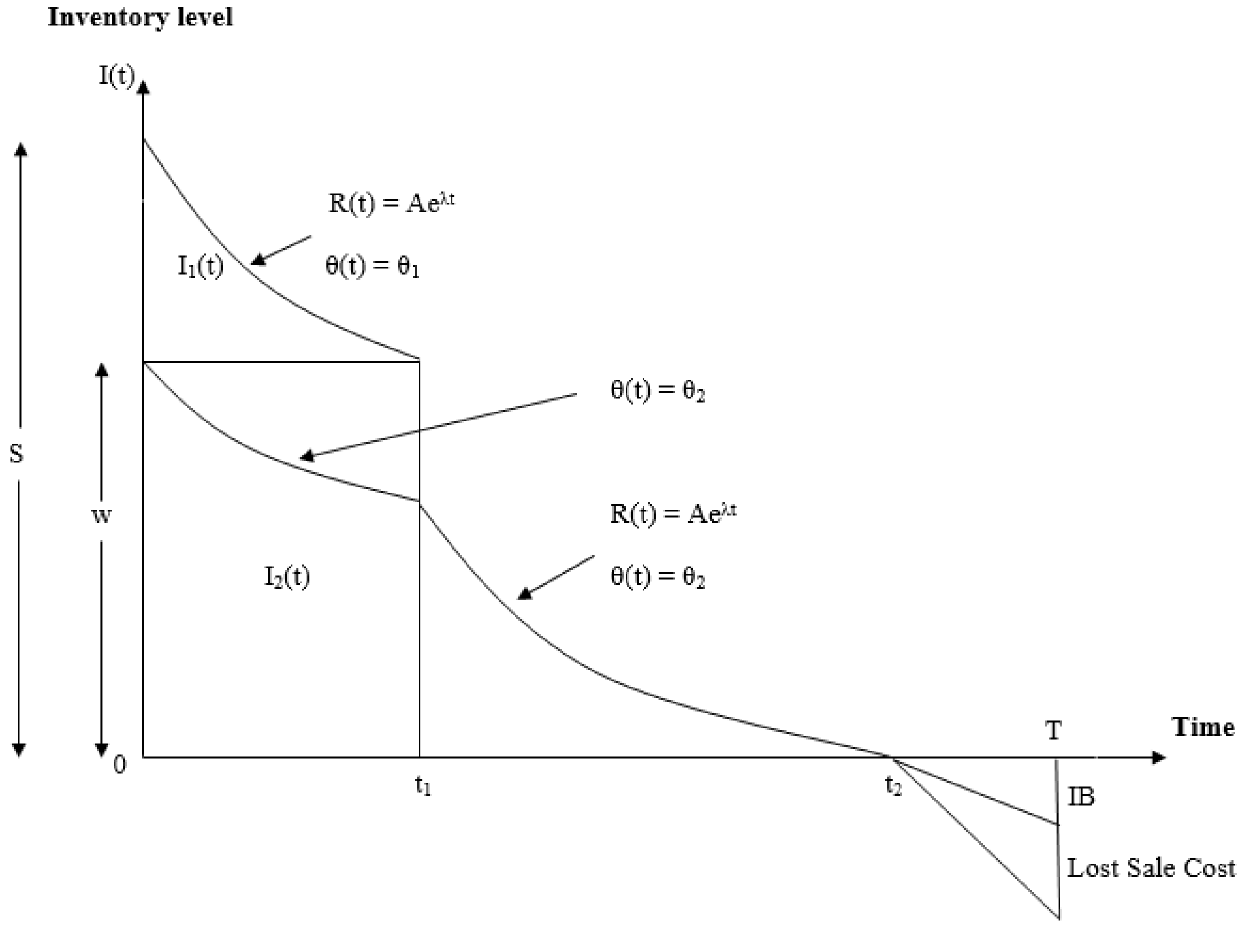

Figure – 1: Graphical presentation of the inventory system

The differential equations which describe the instantaneous state of I(t) over the period (0, T) are given by

$$\frac{dI_1(t)}{dt} + \theta_1 I_1(t) = -Ae^{\lambda t} \qquad (0 \le t \le t_1) \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta_2 I_2(t) = 0 \qquad (0 \le t \le t_2) \tag{2}$$

$$\frac{dI_2(t)}{dt} + \theta_2 I_2(t) = -Ae^{\lambda t} \qquad (t_1 \le t \le t_2) \tag{3}$$

$$\frac{dI_3(t)}{dt} = -Ae^{\lambda t}e^{-\delta(T-t)} \qquad (t_2 \le t \le T) \tag{4}$$

Under the boundary conditions $I_1(t_1) = 0$, $I_2(0) = w$, $I_2(t_2) = 0$, and $I_3(t_2) = 0$, solutions of equations (1) to (4) are given by

$$I_1(t) = \frac{A}{\lambda + \theta_1}\left[e^{(\lambda+\theta_1)\,t_1 - \theta_1 t} - e^{\lambda t}\right] \qquad (0 \le t \le t_1) \tag{5}$$

$$I_2(t) = we^{-\theta_2 t} \qquad (0 \le t \le t_1) \tag{6}$$

$$I_2(t) = \frac{A}{(\lambda+\theta_2)}\left[e^{(\lambda+\theta_2)t_2 - \theta_2 t} - e^{\lambda t}\right] \qquad (t_1 \le t \le t_2) \tag{7}$$

$$I_3(t) = \frac{Ae^{-\delta T}}{\lambda + \delta}\left[e^{(\lambda+\delta)t_2} - e^{(\lambda+\delta)t}\right] \qquad (t_2 \le t \le T) \tag{8}$$

From (6), $I_2(t_1) = we^{-\theta_2 t_1}$ (9)

and from (7), $I_2(t_1) = \frac{A}{(\lambda+\theta_2)}\left[e^{(\lambda+\theta_2)t_2 - \theta_2 t_1} - e^{\lambda t_1}\right]$ (10)

Eliminating $I_2(t_1)$ from equations (9) and (10), we get

$$t_2 = t_1 + \frac{1}{\lambda + \theta_2}\ln\left[\frac{w(\lambda + \theta_2)}{A}e^{-(\lambda+\theta_2)t_1} + 1\right] \tag{11}$$

Thus, t_2 can be written in terms of t_1 and so t_2 is not a decision variable.

The maximum backordered inventory is obtained at t = T and it is denoted by IB. Then from equation (8)

$$IB = -I_3(T)$$

$$\Rightarrow IB = \frac{Ae^{-\delta T}}{\lambda+\delta}\left[e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_2}\right] \quad (12)$$

Maximum inventory level

$$IM = S = I_1(0) + I_2(0)$$

$$\Rightarrow IM = \frac{A}{\lambda+\theta_1}\left[e^{(\lambda+\theta_1)t_1} - 1\right] + w \quad (13)$$

Thus, the order quantity during total interval [0, T] is given by

$$Q = IM + IB$$

$$\Rightarrow Q = \frac{A}{\lambda+\theta_1}\left[e^{(\lambda+\theta_1)t_1} - 1\right] + w + \frac{Ae^{-\delta T}}{\lambda+\delta}\left[e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_2}\right] \quad (14)$$

The total cost comprises of the following costs

1) Ordering Cost

The operating cost (OC) over the period [0, T] is

$$OC = A \quad (15)$$

2) Deterioration Cost

The deterioration cost (DC) over the period [0, t_2] is

$$DC = C_d\left[\int_0^{t_1}\theta_1 I_1(t)dt + \int_0^{t_1}\theta_2 I_2(t)dt + \int_{t_1}^{t_2}\theta_2 I_2(t)dt\right]$$

$$\Rightarrow DC = C_d\left\{\left[\frac{\theta_1 Ae^{\lambda t_1}}{\lambda+\theta_1}\left\{\frac{e^{\theta_1 t_1}-1}{\theta_1} + \frac{e^{-\lambda t_1}-1}{\lambda}\right\}\right] + +w\left(1 - e^{-\theta_2 t_1}\right) + \frac{\theta_2 Ae^{\lambda t_2}}{\lambda+\theta_2}\left[\frac{e^{\theta_2(t_2-t_1)}-1}{\theta_2} + \frac{e^{-\lambda(t_2-t_1)}-1}{\lambda}\right]\right\} \quad (16)$$

3) Inventory Holding Cost

The inventory holding cost (IHC) over the period [0, t_2] is

$$IHC = \int_0^{t_1}(a_2 + b_2 t)I_1(t)dt + \int_0^{t_1}(a_1 + b_1 t)I_2(t)dt + \int_{t_1}^{t_2}(a_1 + b_1 t)I_2(t)dt$$

$$\begin{aligned}\Rightarrow IHC = \frac{Ae^{-\lambda t_1}}{\lambda+\delta}&\left[a_2\left\{\frac{e^{\theta_1 t_1} - 1}{\theta_1} + \frac{e^{-\lambda t_1} - 1}{\lambda}\right\} + b_2\left\{\frac{e^{t_1\theta_1} - t_1\theta_1 - 1}{\theta_1^2} - \frac{e^{-\lambda t_1} + \lambda t_1 - 1}{\lambda^2}\right\}\right] \\ &+ w\left[\frac{a_1\left(1 - e^{-\theta_2 t_1}\right)}{\theta_2} - \frac{b_1\left(t_1\theta_2 e^{-\theta_2 t_1} + e^{-\theta_2 t_1} - 1\right.}{\theta_2^2}\right] \\ &+ \frac{Ae^{\lambda t_2}}{\lambda+\theta_2}\left[a_1\left\{\frac{e^{\theta_2(t_2-t_1)} - 1}{\theta_2} + \frac{e^{-\lambda(t_2-t_1)} - 1}{\lambda}\right\}\right. \\ &\left. + b_1\left\{\frac{t_1 e^{\theta_2(t_2-t_1)} - t_2}{\theta_2} + \frac{e^{\theta_2(t_2-t_1)} - 1}{\theta_2^2} + \frac{t_2 - t_1 e^{-\lambda(t_2-t_1)}}{\lambda} + \frac{e^{-\lambda(t_2-t_1)} - 1}{\lambda^2}\right\}\right]\end{aligned} \quad (17)$$

4) Shortage Cost

The shortage cost (SC) over the period [t_2, T] is

$$SC = -C_S\int_{t_2}^{T} I_3(t)dt$$

$$\Rightarrow SC = \frac{-1}{(\lambda+\delta)^2}\left(C_s Ae^{\lambda t_2 + (t_2-T)\delta}\right)\left[(t_2 - T)(\lambda+\delta) + e^{(\lambda+\delta)(t_2-T)} - 1\right] \quad (18)$$

5) Lost Sale Cost

Opportunity cost due to lost sale (LSC) over the period [t_2, T] is

$$LSC = l\int_{t_2}^{T}(1 - e^{-\delta(T-t)})Ae^{\lambda t}dt$$

$$\Rightarrow LSC = A\,l\left[\frac{e^{\lambda T} - e^{t_2}}{\lambda} - \frac{e^{\lambda T} - e^{(\lambda+\delta)t_2 - \delta T}}{\lambda+\delta}\right] \quad (19)$$

6) Purchase cost

The purchase cost (PC) during the period [0, T] is

$$PC = p_c \cdot Q$$

$$\Rightarrow PC = p_c\left\{\frac{A}{\lambda+\theta_1}\left[e^{(\lambda+\theta_1)t_1} - 1\right] + w + \frac{Ae^{-\delta T}}{\lambda+\delta}\left[e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_2}\right]\right\} \quad (20)$$

Hence the total cost per unit time is given by

$$TC = \frac{1}{T}[OC + DC + IHC + SC + LSC + PC] \quad (21)$$

Substituting the values of OC, IHC, DC, SC, LSC and PC from equations (15) to (20), we get

$$
=\frac{1}{T}\left\{\begin{aligned}
&A+C_d\left\{\begin{aligned}&\left[\frac{\theta_1 A e^{\lambda t_1}}{\lambda\theta_1}\left\{\frac{e^{\theta_1 t_1}-1}{\theta_1}+\frac{e^{-\lambda t_1}-1}{\lambda}\right\}\right]+\left[w\left(1-e^{-\theta_2 t_1}\right)\right]\\&+\frac{\theta_2 A e^{\lambda t_2}}{\lambda+\theta_2}\left[\frac{e^{\theta_2(t_2-t_1)}-1}{\theta_2}+\frac{e^{-\lambda(t_2-t_1)}-1}{\lambda}\right]\end{aligned}\right\}\\
&+\frac{Ae^{-\lambda t_1}}{\lambda+\delta}\left[\begin{aligned}&a_2\left\{\frac{e^{\theta_1 t_1}-1}{\theta_1}+\frac{e^{-\lambda t_1}-1}{\lambda}\right\}\\&+b_2\left\{\frac{e^{t_1\theta_1}-t_1\theta_1-1}{\theta_1^2}-\frac{e^{-\lambda t_1}+\lambda t_1-1}{\lambda^2}\right\}\end{aligned}\right]\\
&+w\left[\frac{a_1\left(1-e^{-\theta_2 t_1}\right)}{\theta_2}-\frac{b_1(t_1\theta_2 e^{-\theta_2 t_1}+e^{-\theta_2 t_1}-1}{\theta_2^2}\right]\\
&+\frac{Ae^{\lambda t_2}}{\lambda+\theta_2}\left[\begin{aligned}&a_1\left\{\frac{e^{\theta_2(t_2-t_1)}-1}{\theta_2}+\frac{e^{-\lambda(t_2-t_1)}-1}{\lambda}\right\}\\&+b_1\left\{\frac{t_1e^{\theta_2(t_2-t_1)}-t_2}{\theta_2}+\frac{e^{\theta_2(t_2-t_1)}-1}{\theta_2^2}+\frac{t_2-t_1e^{-\lambda(t_2-t_1)}}{\lambda}+\frac{e^{-\lambda(t_2-t_1)}-1}{\lambda^2}\right\}\end{aligned}\right]\\
&-\frac{C_s A e^{\lambda t_2+(t_2-T)\delta}}{(\lambda+\delta)^2}\left[(t_2-T)(\lambda+\delta)+e^{(\lambda+\delta)(t_2-T)}-1\right]+Al\left[\frac{e^{\lambda T}-e^{t_2}}{\lambda}-\frac{e^{\lambda T}-e^{(\lambda+\delta)t_2-\delta T}}{\lambda+\delta}\right]\\
&+p_c\left\{\frac{A}{\lambda+\theta_1}\left[e^{(\lambda+\theta_1)t_1}-1\right]+w+\frac{Ae^{-\delta T}}{\lambda+\delta}\left[e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_2}\right]\right\}
\end{aligned}\right\} \tag{22}
$$

Our objective is to determine optimum values t_1* and T* of t_1 and T respectively so that cost function TC is minimum. Note that t_1* and T* are the solutions of the equations $\frac{\partial C}{\partial t_1}=0$, $\frac{\partial C}{\partial T}=0$ which can satisfy the following sufficient conditions:

$$
\left.\begin{aligned}
&\left|\begin{matrix}\frac{\partial^2 TC}{\partial^2 t_1} & \frac{\partial^2 TC}{\partial t_1\partial T}\\ \frac{\partial^2 TC}{\partial t_1\partial T} & \frac{\partial^2 TC}{\partial^2 T}\end{matrix}\right|_{t_1=t_1^*,\ T=T^*} > 0\\
&\text{and } \left.\frac{\partial^2 TC}{\partial^2 t_1}\right|_{t_1=t_1^*,\ T=T^*} > 0
\end{aligned}\right\} \tag{23}
$$

The optimal values t_1* and T* can be obtained by using Maple software.

The above developed model is illustrated by means of the following numerical example.

V. Numerical Example

To illustrate the proposed model, an inventory system with the following hypothetical values is considered. By taking C = 200, $\theta_1 = 0.1$, $\theta_2 = 0.06$, $\lambda = 0.3$, $\delta = 0.2$, w = 80, A = 50, Cs = 3, $C_p = 20$, $C_d = 5$, $\ell = 8$, $a_1 = 1$, $b_1 = 0.05$, $a_2 = 3$ and $b_2 = 0.06$ (with appropriate units).

The optimal values of t_1 and T are t_1* = 0.8142968570, T* = 1.626137857 units and the optimal total cost per unit time TC = 1618.688443 units.

VI. Sensitivity Analysis

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, we study the sensitivity for the total cost per time unit TC with respect to the changes in the values of the parameters C, θ_1, θ_2, λ, δ, w, A, Cs, C_p, C_d, ℓ, a_1, b_1, a_2 and b_2. The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other remaining parameters as fixed.

Table 1. Partial Sensitivity Analysis

Parameter	Values	t_1	T	TC
C	120	0.769829434	1.529050097	1567.984900
	160	0.792734638	1.578647939	1593.726540
	240	0.834647565	1.671730355	1642.945872
	280	0.853897867	1.715603694	1666.562682
θ_1	0.083	1.241955356	1.652027823	1609.851299
	0.084	1.200598189	1.648753400	1610.665987
	0.085	1.162736679	1.645910817	1611.421594
	0.090	1.011115311	1.635946741	1614.526234
θ_2	0.065	0.522702509	1.604312712	1618.301281
	0.070	0.350074966	1.588972539	1615.508980
	0.075	0.230624613	1.576666533	1612.240555
	0.080	0.139972356	1.566423269	1609.080601
Λ	0.4	0.394351701	1.333439556	1699.053230
	0.5	0.279864001	1.146702190	1765.226664
	0.6	0.229701080	1.013777501	1824.497877
	0.7	0.203121387	0.913283076	1878.957615
Δ	0.23	0.858395804	1.617057090	1618.314359
	0.25	0.900374509	1.608492930	1617.952493
	0.28	0.999207757	1.588649049	1617.075700
	0.30	1.112317225	1.566513194	1616.020596
W	81	0.847575392	1.634360751	1621.048511
	82	0.880670077	1.642553637	1623.354659
	84	0.946298125	1.658851742	1627.806118
	85	0.978827386	1.666958126	1629.951935
A	37	1.720957413	1.912311584	1260.104399
	41	1.397400519	1.810299238	1375.179031
	47	0.989330354	1.681954847	1539.531114
	49	0.870710940	1.644218132	1592.499708
Cs	1.8	1.219382565	1.546648081	1615.404235
	2.0	1.053812585	1.578032409	1616.710476
	2.4	0.908433989	1.606883969	1617.906867
	2.8	0.837902342	1.621270422	1618.492673
p_c	24	0.513813346	1.514095788	1891.158147
	26	0.383902479	1.473456736	2024.473817
	28	0.259515014	1.440820922	2156.246247
	30	0.135199227	1.415615024	2286.669715
C_d	2.0	1.155111390	1.626071716	1600.639217
	3.0	1.024553525	1.624541623	1607.420405
	4.0	0.912334740	1.624785241	1613.378829
	5.5	0.769661775	1.627098798	1621.140590
ℓ	4.8	0.913944682	1.605758203	1617.849915
	5.5	0.884330024	1.611766627	1618.100679
	6.0	0.866587304	1.615386218	1618.250309
	9.0	0.795586252	1.630014825	1618.844098
a_1	0.89	0.697311306	1.619906974	1620.165136
	0.90	0.708278031	1.620516862	1620.063309
	0.92	0.730005822	1.621709964	1619.839842
	0.93	0.740768620	1.622293329	1619.718294
Parameter	**Values**	**t_1**	**T**	**TC**
b_1	0.040	0.425917770	1.563254513	1599.392780
	0.044	0.538446447	1.587147617	1608.375182
	0.045	0.573176351	1.593235892	1610.418171
	0.046	0.611398081	1.599417611	1612.357662
	2.90	0.874998705	1.628735077	1617.352236

a_2	3.10	0.761955008	1.624123138	1619.838064
	3.15	0.738369305	1.623281229	1620.355070
	3.20	0.716276179	1.622528918	1620.838673
b_2	0.050	0.816580163	1.626259740	1618.654325
	0.055	0.815435207	1.626198552	1618.671423
	0.065	0.813165050	1.626077647	1618.705389
	0.070	0.812039709	1.626017919	1618.722259

VII. Graphical Presentation

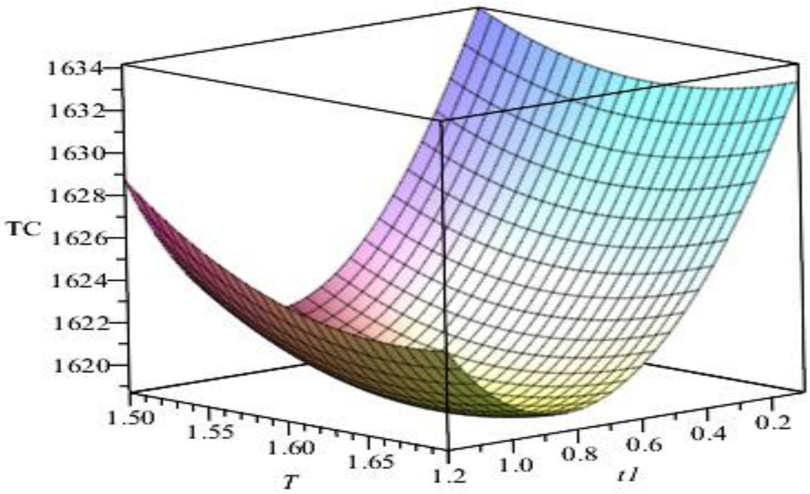

Figure 2.

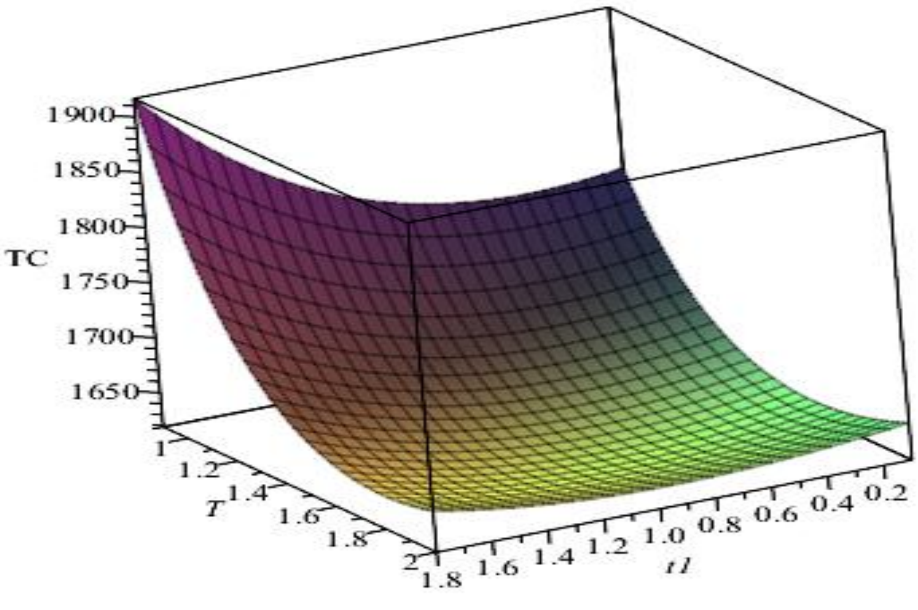

Figure 3.

VIII. Conclusions

1. From the Table – 1, we observe that as the values of the parameters C, θ_1, λ, w, A, Cs, C_p, C_d, a_1, b_1, a_2 and b_2 increase the average total cost also increases and as the values of the parameters θ_2, δ, ℓ and a_1 increase the average total cost decreases.
2. From the Table – 1, we also observe that that the total cost per time unit is highly sensitive to changes in the values of C, λ, A, and C_p.
3. One can note from Table – 1 that that the total cost per time unit is moderately sensitive to changes in the values of b, θ_2, w, C_d, and b_1.
4. Also, from same table, we note that the total cost per time unit is less sensitive to changes in the values of θ_1, δ, Cs, ℓ , a_1, a_2 and b_2.
5. Figure – 2 and Figure – 3 show the effect of decision variables t_1 and T on average total cost TC.

IX. References

1. Ajay Singh Yadav, Anupam Swami (2013), A Two-warehouse Inventory Model for Decaying Items with Exponential Demand and Variable Holding Cost, *International Journal of Inventive, Engineering and Sciences, Vol. 1*, Issue 5.
2. Benkherouf (1997), A Deterministic Order Level Inventory Model for Deteriorating Items with Two Storage Facilities, *International Journal of Production Economics, 48*, 167-175.
3. Bhojak A. and Gothi U. B. (2015), Inventory Model for Ameliorating and Deteriorating Items with Time Dependent Demand. *Sankhya Vignan, (NSV 11), No.2*, 31-43.
4. Buzacott J. A. (1975), Economic order quantities with inflation, *Operational Research Quarterly, Vol 26, no. 3*, 553-558.
5. Chatterji, D. A. and Gothi, U. B. (2015), Three-parametric Weibully deteriorated EOQ model with price dependent demand and shortages under fully backlogged condition, *International Journal of Innovative Research in Science, Engineering and Technology, Vol. 4, Issue 12*, 12710-12720.
6. Dave U. (1988), On the EOQ models with two levels of storage, *Opsearch, 25*, 190-196.
7. Dave U. (1989), A Comment on "Inventory replenishment policy for linearly increasing demand considering Shortages – An optimal Solution by T. N. Murdeshwar, *JORS, 40*, 412-416.
8. Donaldson W. A. (1977), Inventory replenishment policy for a linear trend in demand – an analytical solution, *OR (Quart.), 28*, 663-670.
9. Goswami and Chaudhuri (1992), An economic order quantity model for items with two levels of storage for linear trend in demand, *Journal of Operational Research Society, 43*, 157-167.
10. Hariga M. A. (1995), Effects of inflation and time-value of money on an inventory model with time-dependent demand rate and shortages, *European Journal of Operational Research, Vol. 81, no. 3*, 512-520.
11. Hartley, R.V., (1976) Operation Research — A Managerial Emphasis, *Good Year Pub. Comp*, 315–317.
12. Hsieh, Dye and Ouyang (2008), Determining optimal size for a two-warehouse system with deterioration and shortages using net present value, *European Journal of Operational Research, 191*, 182-192.
13. Jaggi et al. (2007), optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy, *European Journal of Operational Research, Vol. 176, Issue 3*, 1577-1591.
14. Kar S., Bhunia A. K. and Maiti M. (2001), Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon, *Computers and Oper. Res., Vol. 28*, 1315-1331.
15. Kumar et. al. (2013), an inventory model with time dependent demand and limited storage facility under inflation, *Advances in Operations Research, Vol. 2012, Article ID 321471*.
16. Misra R. B. (1975), a study of inflationary effects on inventory systems, *Logistic spectrum, Vol. 9*, 260-268
17. Murdeshwar T. M. and Sathe Y. S. (1985), some aspects of lot size models with levels of storage, *22*, 255-262.
18. Pakkala and Achary (1992), A deterministic inventory model for deteriorating items with two-warehouses and finite replenishment rate, *European Journal of Operational Research, 57*, 71-76.
19. Parmar, K., Gothi U. B. (2015), EOQ model with constant deterioration rate and time dependent demand and IHC", *Pacific Business Review International, Vol. 7, Issue 12*.
20. Sarma K. V. S. (1987), A Deterministic Order-level Inventory Model for Deteriorating Items with Two Storage Facilities, *European Journal of Operational Research, 29*, 70-72.
21. Yang H. L. (2004), Two-Warehouse Inventory models for Deteriorating Items with Shortages under Inflation, *European Journal of Operational Research, 157*, 344-356.
22. Zhou Y. W. and Yang S. L. (2005), A Two-warehouse Inventory Model for Items with Stock-Level-Dependent demand rate, *International Journal of Production Economics*, 95, 215-228.