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Understanding Avalanche Statistics on Crack Dynamics

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Abstract

In brittle solids, stress concentration at crack tips makes the macroscopic fracture properties very sensitive to heterogeneities at the scale of the micro structure. So, the macroscopic resistance of a solid depends strongly on the resistance fluctuations at the microscopic scale. To describe quantitatively this phenomenon in disordered materials, we model first the behavior of the crack by a stochastic equation of motion taking into account the role of the microstructure. Our approach is first validated by comparing our theoretical predictions with recent experimental observations made on the dynamics and morphology of a crack front. We show how to use this approach to determine the effective resistance of a brittle material from the characteristics of its micro structure.

Keywords: Heterogeneous Materials; Dynamic Cracks, Effective Resistance.

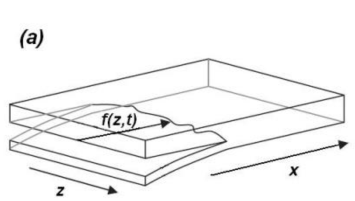
Introduction

Breakage of heterogeneous materials is of great importance in many areas. This subject is far from understood, and has been widely studied in recent decades. The competition between the trapping forces due to the heterogeneity of the material and the elastic forces due to the loading plays a central role for fragile materials, and governs largely the crack behavior. Several works, both experimental and theoretical, have been devoted to the behavior of coplanar interfacial fissures in disordered materials. In this type of system, it has been established that the crack front has scale invariance properties. Because of the heterogeneity of the material, the forehead is spread by sudden leaps of all sizes. At first, we will recall the foundations of our approach to writing the movement of a crack in a heterogeneous interface. We will then test the relevance of our approach by comparing our theoretical predictions with recent experimental observations made in such a geometry [1]. We will then use our model to determine how the characteristics of the heterogeneous field of resistance at the microscopic scale affect the macroscopic effective resistance of the interface.

Methodology

Equation of evolution of the crack front

The geometry of the studied system, is inspired by the experimental device of [Ref] which is schematically presented by the diagram Figure I. An interface crack of length f (z, t) is propagates through the interface of two elastic plates separated by an opening length d. All characteristic lengths are assumed to be significantly larger than the forehead fluctuations and the characteristic length of heterogeneities. Thus, the 3D problem of coplanar crack propagation in a brittle material, can be reduced to a 2D problem where the crack front is moving in the plane of the interface with heterogeneous breaking properties.



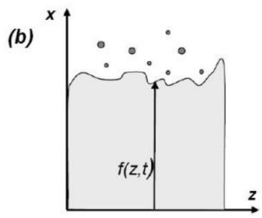


Figure 1 *Figure Ia*
Interface Crack

Whereμrepresents the mobility of the front and$G(z,t) =$ $G_0 + k\big(v_m - f(z,t)\big) + \frac{G_0}{\pi}\int_{-\infty}^{+\infty}\frac{f(z')-f(z)}{(z'-z)^2}dz'$. In the case of a stable propagation of the crack considered here, k is a positive constant dependent of the geometry of the sample and v_m is the average speed imposed on the crack by the external loading. G_0corresponds to the macroscopic restitution rate imposed on the system. We then obtain the evolution of the crack front at the disordered interface by numerically solving equation (1).

Now let's go to the description of the breaking properties of our model. We begin by recalling the experimental procedure followed to prepare the sample

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shown in FIG. I (a). Before gluing the two plexiglass plates together with a treatment thermal, one of the surfaces is sandblasted so that the interface is reinforced heterogeneously. This introduces variations in the fracture properties that we describe by a G c field. (x, z), called breaking energy.

We then assume that this field is characterized by a correlation length ξ which corresponds to the typical size of the possibly linked heterogeneities the diameter of the grains used for sanding [21]. This field of heterogeneity is then pulled into a Gaussian distribution of mean value G c and the standard deviation δG c. The heterogeneity of the material generates geometric disturbances of the crack front. These then generate variations in the rate of energy restitutions (,) along the front. In the the limit of weak disturbances, this variation (,) can be calculated by the formula of Rice [3]. The equation of motion is then obtained using the fact that the local velocity of front is proportional to the difference between the rate of energy restitution and the energy of rupture[4]:

$$\frac{1}{\mu}\frac{\partial f}{\partial t} = G(z,t) - G_c(z, x = f(z,t)) \quad (1)$$

Numerical resolution of the crack front evolution equation

To predict the dynamics of the crack front, we focus on the scaled evolution equation (1) and follow the numerical procedure used by Bonamy et al.[9], this equation is strongly nonlinear because of the presence of the integral term. To solve it, we go through a numerical resolution. This method consists in discretizing the front in Nz elements where Nz represents the size of the system in the direction along the front and Nx that corresponds to the direction of propagation. At a given time t, the configuration of the front is described by Nzvalues $\{f_1(t), f_2(t), \cdots, f_{Nz}(t)\}$. We have also imposed periodic boundary conditions along the z axis. For each numerical simulation, we extract three quantities that will be used later for the statistical characterization of the forehead dynamics:

- The spatio-temporal evolution is stored in the matrix $(f_i(t_j))$.
- The local velocity of the crack front is stored in the matrix (v_{ij}).
- The time spent by each point of the forehead is stored in the matrix (w_{ij}).

Results and discussion

In order to validate our approach, we compare the predictions of our model on the statistical properties of the front with experimental observations [4], [5].

Roughness of the crack front

From our model, we have numerically calculated the autocorrelation function of height fluctuations (Fig. II) defined by:

$$\Delta f(\delta z) = < [f(z + \delta z) - f(z,t)] >_{z,t}^{1/2} \quad (2)$$

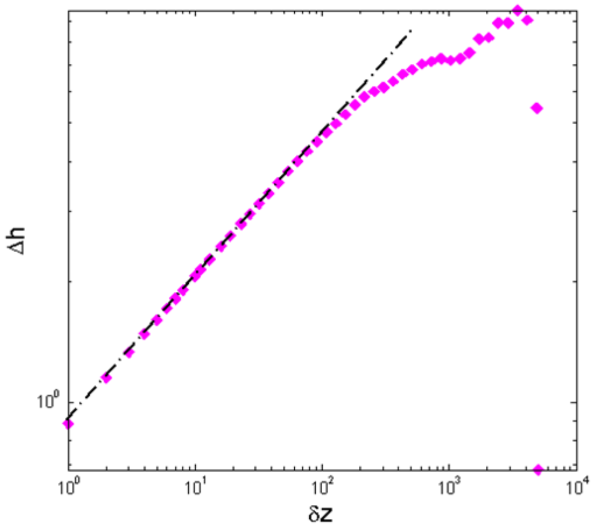

Figure II
Correlation function of geometric disturbances of the crack front. We observe that this function follows a power law$\Delta f\,(\delta z) \propto \delta z^{\zeta}and\zeta \sim 0.4$.

We observe that this function follows a power This autocorrelation function is characterized by the difference in height corresponding to the difference in height between two points distant from the front. law$\Delta f\,(\delta z) \propto \delta z^{\zeta}and\zeta \sim 0.4$. This result is in agreement with the theoretical predictions for disordered elastic line trapping equations similar to equation (1) [5]. It is also in agreement with the experimental observations made on cracks propagating at the heterogeneous interface between two Plexiglas plates [1].

Correlation of speeds

To characterize the local dynamics of the front, we adopt a method of analysis developed in [6] to calculate the time spent by the front at each point of the interface. From this method, we obtain a waiting time matrix w (z, x) to have the local velocities v (z, t) along the front. We can then calculate the autocorrelation function of local velocities in time (Figure III). Function follows an exponential law of form$C(\delta t) \propto e^{-\delta t/t^*}$with$t^* \sim \frac{1}{v}$. These results are in good agreement with recent observations [1].

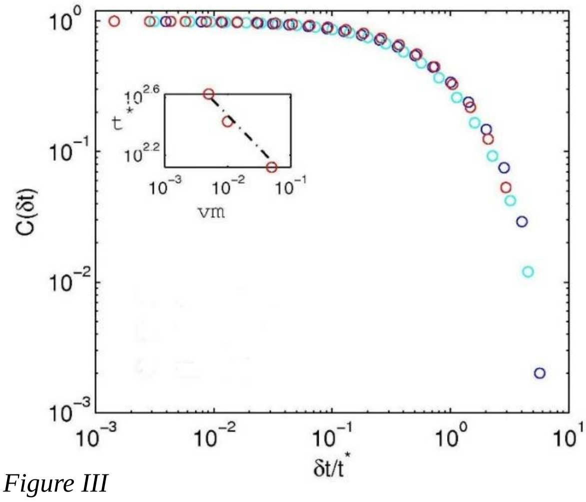

Figure III
Standardized speed correlation functionC(δt)

On the other hand, we observe that the local dynamics of the front are very intermittent, characterized by rapid jumps from the front from one position to another, also called avalanches. In the next section, we study the statistics of these avalanches.

Statistical properties of avalanches

We define avalanches as areas of surface S of the interface where the local velocity of the front is greater than a threshold value C. In Figure IVa, the size distribution of these avalanches is represented for different threshold values C It follows the following law:

$$P(S) \sim S^{\gamma} exp(-S/S^{*}) \qquad (3)$$

After normalization of the curves by the quantity$S^{*}(c)$, we obtain the evolution given in Figure. IVb, characterized by the exponent $\gamma \sim 1.6$. in good agreement with the experiments [6].

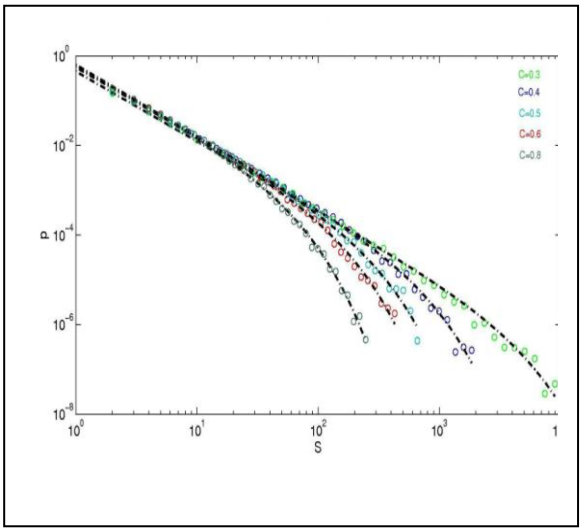

Figure IVa

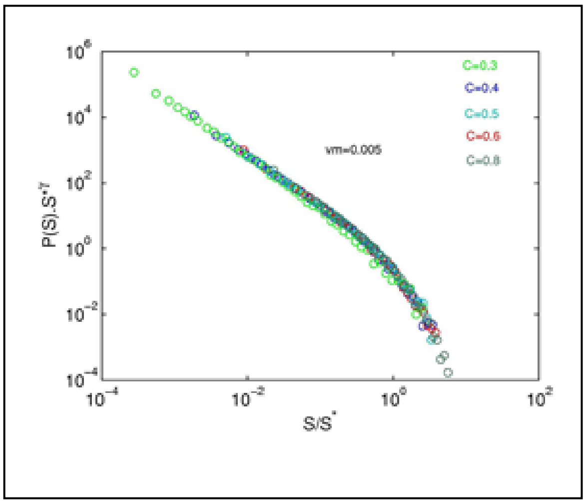

Figure IVb
Avalanche size distribution before and after normalization

Application to the prediction of the effective resistance of disordered solids

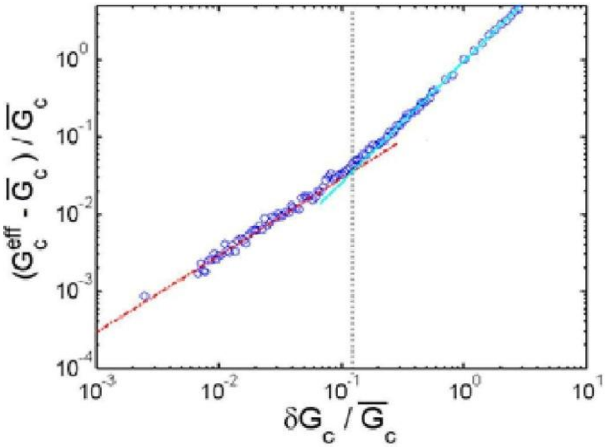

Figure V
Variation in effective macroscopic resistance as a function of resistance fluctuations at the microscopic scale

As illustrated previously, our approach allows to describe the effect of the microstructure on the behavior of the crack. We will then use this approach to identify the properties of the microstructure of the material that govern its effective macroscopic resistance. Figure 5 shows the predictions for normalized breaking energy variations$\frac{(G_c - \langle G_c \rangle)}{\langle G_c \rangle}$by its average value$\langle G_c \rangle = \overline{G_c}$a disordered fragile interface according to its standardized standard deviation$\sigma_{Gc} = \delta G_c / \langle G_c \rangle$.

We observe two regimes:
–a linear regime for which $G_c \sim \sigma_{Gc}$for$\sigma_{Gc} < 0.1$
– and a regime in power law$G_c \sim \sigma_{G_c}^{\alpha}for\sigma Gc > 0.1$with$\alpha \sim 1.6$

In this case, the more material disorder increases significantly, the higher the effective resistance of the material interface.

Conclusion

From the stochastic description proposed here, it is possible to quantitatively describe the motion of a crack front in a disordered interface. This approach has also allowed us to predict the effect of the disorder of the interface on its effective resistance. In particular, it is observed that the effective resistance increases significantly with the disorder of the material. This suggests that the heterogeneities of solids at the microstructure level could be used to increase their strength.

Reference

1. K.T. Tallaksatad, R. Toussaint, S. Santucci, J. Schmittbuhl and K.J. Maloy, Local Dynamics of a Randomly Pinned Crack Front during Creep and Forced Propagation : An Experimental Study, Phys.Rev. E, 83:046108, 2011
2. S. Santucci, Grob M., R. Toussaint, S. Santucci, J. Schmittbuhl, A. Hansen and K.J. Maloy, Fractureroughness scaling : A case study on planar cracks, EPL 92, 571-579, 2010.
3. J.R. Rice, First-order variation in elastic fields due to variation in location of a planar crack front. J. of Appl. Mech. 52, 571-579, 1985.
4. D. Bonamy and L. Ponson, Crackling dynamics in material failure as the signature of a self-organized dynamic phase transition, Phys. Rev. Lett 101, 045501, 2008.
5. A. Rosso and W. Krauth, Roughness at the depinning threshold for a long-range elastic string, Phys Rev E 65, 025101, 2002.
6. K.J. Maloy, S. Santucci, J. Schmittbuhl and R. Toussaint, Local Waiting Time Fluctuations along a Randomly Pinned Crack Front , Phys. Rev. Lett. 96, 045501, 2006.